Field correlators in QCD.
Theory and applications.

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Abstract

This review is aimed to demonstrate the basics and use of formalism of gauge-invariant nonlocal correlators in nonabelian gauge theories. Many phenomenologically interesting nonperturbative aspects of gluodynamics and QCD can be described in terms of correlators of the nonabelian field strength tensors. It is explained how the properties of correlator ensemble encode the structure of QCD vacuum and determine different nonperturbative observables. It is argued that in gluodynamics and QCD the dominant role is played by the lowest nontrivial two-point correlator (Gaussian dominance). Lattice measurements of field correlators are discussed. Important for the formalism theoretical tools, such as nonabelian Stokes theorem, background perturbation theory, cluster expansion, as well as phenomenological applications to the heavy quarkonium dynamics and QCD phase transition are reviewed.

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Contents

1 Introduction and motivation

This review is devoted to the study of gauge theories, specifically gluodynamics and QCD_4. There are now many good books on QCD [1, 2, 3, 4], which mainly concentrate on perturbation theory. The nonperturbative (NP) aspects of QCD (and gauge theories in general) have a somewhat peculiar status. On the one hand our very existence is based on one of the most important NP phenomenon – confinement. On the other hand perturbative QCD disregards confinement and treats quarks and gluons as weakly interacting. Perturbative QCD is a vast field of investigations which makes most of the contents of the QCD textbooks leaving for the NP part of QCD only few short qualitative chapters at the end.

The main unsolved problem of QCD is the quantitative treatment of NP phenomena. We still do not have selfconsistent formalism where nonperturbative quantities, like masses and condensates, are computed systematically from first principles, i.e. from the QCD Lagrangian. Nowadays the only reliable way of getting NP results in QCD is numerical simulation of the theory on the lattice. This area of research is of great importance, bringing many insights into the subject [5, 6].

Another source of the NP intuition in QCD is the analysis of quantum field theoretical models simplified in one respect or another. These are a huge zoo of low-dimensional models (notably QCD_2), abelian systems, supersymmetric theories and many others. The studies of the corresponding exact solutions provide substantial information relevant for physically more interesting cases.

We have no experimental chance to study the properties of the world without light quarks. However, lattice calculations and theoretical analysis in the limits where quark degrees of freedom are decoupled show, that the property of confinement (i.e. absence of colored particles as asymptotic states [7]) holds in the theory without quarks (pure gluodynamics) as well as in real QCD. It suggests to study the physics of confinement starting from quenched approximation, as is done, for example, in $1/N_c$ – expansion. The asymptotic freedom [8, 9] and dimensional transmutation phenomena provide the NP mass scale, Λ_{QCD} and force the theory to leave its perturbative domain at large distances. All other dimensionful quantities (such as string tension for static external sources, gluon condensate, glueball and hybrid masses and widths) in the quenched approximation are proportional to corresponding power of Λ_{QCD} with some dimensionless coefficients that in principle can be computed in NP QCD.

The chiral effects, which are mostly responsible for physics of lightest hadrons, are originated from the NP gluonic sector of the theory as well since there is no spontaneous chiral symmetry breaking in perturbative QCD. The confinement – chiral symmetry breaking correspondence, however, is very far from trivial. We do not touch this set of questions in this review, planning to present an extended separate discussion of this subject elsewhere. An interested reader can find a lot about the light quark physics in the confining vacuum in the gauge-invariant field correlators approach in [10].

An important question is related with possible phases of the theory and, in particular, with the NP effects in different phases. Internal parameters of the theory (such as number of colours and flavours, light quark masses) and external conditions (temperature, chemical potential, number of space-time dimensions) both play important role here. They determine the symmetry of the ground state at a given point of the phase diagram and corresponding spectrum of light excitations. Each point of the phase diagram can be characterized by the values of condensates, describing the symmetry breaking pattern. The simplest condensates are given by the NP gluon and quark condensates:

$$\langle \frac{\alpha_s}{\pi} F_{\mu\nu} F_{\mu\nu} \rangle \quad ; \quad \langle \bar{\psi}\psi \rangle \tag{1.1}$$

The average in (1.1) is defined by (2.5) below and all perturbative contributions are assumed to be subtracted from (1.1). The NP–generated nonzero value of the gluon condensate [11] corresponds to the breaking of scale invariance at the quantum level (notice, that finite dimensionful parameter cannot be generated perturbatively). Nonzero value of the quark condensate indicates chiral symmetry breaking [1, 4]. One can also consider different higher order operators as well as nonlocal averages. In the high density – low temperature limit the nonzero diquark condensate $\langle \psi\psi \rangle$ is predicted, leading to the phenomenon of colour superconductivity, while chiral symmetry is restored [12, 13, 14]. The actual values of the condensates also depend on the temperature and another parameters of the theory.

As it has already been mentioned, physics of light quarks will not be discussed in the present review. Our main interest is concentrated on a systematic method of computing NP effects in the quenched approximation from some fundamental NP input, one and the same for all processes. We have chosen for this NP input a set of field strength correlators (FC), namely,

$$\Delta_{\mu_1\nu_1,...,\mu_n\nu_n} = \mathrm{Tr}\, \langle F_{\mu_1\nu_1}(x_1)\Phi(x_1x_2)F_{\mu_2\nu_2}(x_2)...F_{\mu_n\nu_n}(x_n)\Phi(x_nx_1) \rangle \tag{1.2}$$

where $F_{\mu\nu}$'s are the field strength tensors and $\Phi(x, y)$ are the phase factors, introduced for the sake of gauge invariance and discussed in details below.

The systematic use of gauge-invariant variables was adopted for the first time by S.Mandelstam in his celebrated paper [15]. It was shown, that usual Feynman rules of perturbation theory can be recovered from the Dyson-Schwinger equations for the averages (1.2). The key idea proposed in [16, 17, 18] is to use the gauge-invariant quantities (1.2) as a dynamical input also in the nonperturbative domain and to express gauge-invariant observables through (1.2) via the cluster expansion. Research activity during the last 10 years has shown that several QCD processes of phenomenological interest can be calculated using the set (1.2) as an input. More than that, a systematic cluster expansion can be performed, and the first term, corresponding to the FC with $n = 2$ (sometimes called later the Gaussian or bilocal correlator) already gives a good qualitative description of most NP phenomena, while higher cumulants can be considered as corrections (see Section 2). There are evidences that these corrections in the known cases are not large and contribute around a few percents of the total effect. Thus one obtains a theory with a simple but fundamental input – the Gaussian correlator – and the corresponding formalism is called the Gaussian dominance approximation or sometimes the Gaussian stochastic model of QCD vacuum.

The method discussed in the review can be called "fundamental phenomenology" since it uses correlators (actually the lowest Gaussian one) (1.2) as the only dynamical input. The latter is usually given by lattice measurements. It is worth mentioning that actual profile of the Gaussian correlator plays no essential role for the calculation of hadron properties. The necessary NP information enters via string tension σ, which is an integral characteristic of Gaussian correlator and thus one has a unique opportunity to define the hadron spectrum in terms of one parameter. Note also in this respect, that we do not try to reconcile the present approach with any of the particular NP QCD vacuum microscopical models, in particular, instanton liquid model in this review (see e.g. [19]). As it will be clear from what follows, since the quantities (1.2) are eventually the main input objects, one might not in principle be interested what kind of field configurations play the dominant role for the particular behaviour of correlators. On the other hand, most of the dynamical statements of a particularly chosen model about the NP content of the QCD vacuum can typically be reformulated as a gauge–invariant statements about properties of the correlators (1.2), especially if the model provides some recipee to calculate them nonperturbatively (see

also discussion in [20]).

Lattice measurement is a very important source of information on FC. First calculation of the Gaussian FC in [21, 22] was followed by more detailed investigation in [24, 25, 26] where also nonzero temperature was explored. Both Gaussian and quartic FC have been measured while testing the field distribution inside the QCD string [27, 28, 29] and this analysis together with independent measurement done in [30] supports the Gaussian dominance picture.

One should say a few words about the symmetries of QCD vacuum, which make it natural to use FC given by (1.2). Since the fields A_μ as well as $\vec{E}_i$ and $\vec{B}_i$ are vector–like and Lorentz invariance and color neutrality of the vacuum lead to inevitable conclusion that there is no any preferred direction either in the coordinate or in the color space in the physical vacuum of QCD, one immediately gets

$$\langle F_{\mu\nu}(x)\rangle \equiv 0 \tag{1.3}$$

This rules out models with constant vector fields. There is an important difference with the scalar field condensation in Higgs-like theories. The latter is usually characterized by some constant parameter, e.g. value of the scalar field in unitary gauge. This is impossible for the vector field condensation since the construction of true vacuum state implies an averaging procedure over different field configurations in order to get Lorentz and gauge invariant ground state. These averaging algorithms, however, may be of different types: for example, in models based on ensemble of classical field configurations like instanton gas model one performs it by summing over all positions and colour orientations of individual instantons, while in the original quantum case the summation over properly weighted gauge field configurations is "built in" from the beginning in the functional integral approach.

It is known already for more than 20 years [11] that the QCD vacuum is filled with strong NP chromoelectric (CE) and chromomagnetic (CM) fields, which due to the scale anomaly cause the NP shift of the energy density of the vacuum [1]

$$\varepsilon = \frac{\beta(\alpha_s)}{16\alpha_s}\langle F_{\mu\nu}(0)F_{\mu\nu}(0)\rangle \tag{1.4}$$

Because of the asymptotic freedom $\beta(\alpha_s)$ is negative (at least for small α_s) thus making the NP shift ε energetically favorable.

Now a crucial question arises, what are the typical scales of the fields, contributing to the r.h.s. of (1.4)? The original proposal made in [11] con-

sidered these fields as slowly varying in space-time and hence, in the operator product expansion (OPE) formalism the condensates enter as constant coefficients in front of the terms with powers of momentum while higher twist contributions are suppressed as subleading corrections.

However, in the course of lattice simulations it was found [21, 22], that typical correlation length T_g, characterizing the spatial dependence of gauge-invariant bilocal correlator (1.2) is very small, of the order of 0.2-0.3 fm (see discussion below). It seems therefore very natural to expect the onset of conventional OPE at such large Q^2 that $Q^2 >> T_g^{-2}$, while at $Q^2 \simeq T_g^{-2}$ the system crucially feels the nontrivial space-time distribution of the fields and usual OPE can break down.

One can easily see that the method of field correlators (MFC) described in the review is a natural development of the QCD sum–rule method, where the NP inputs are condensates, i.e. vacuum averages of operator products taken at one point. The sum–rule method has natural limits and cannot be applied to long distance processes, since information stored in local averages is not enough to describe dynamics in the region where stochastic nature of the vacuum plays the most important role. Thus higher excited hadron states, string–like spectrum and all properties connected to confinement are outside of realm of the QCD sum–rule method.

Another essential ingredient of the method is the idea of Gaussian dominance. As we shall discuss in Section 3, the Gaussian approximation is a very specific model, where all higher correlators (1.2) are expressible through the product of lowest Gaussian ones. Then all correlators with odd n are assumed to vanish. A less restrictive approximation is the one where the correlators with $n = 2, 3$ are nonzero and the higher FC factorize into product of lowest ones. This approximation is sometimes called the extended Gaussian model. It is worth noting that the term "Gaussian" should not be misunderstood. The theory under consideration is not a free theory. The n - point Green function is assumed to be factorized as a product of bilocal ones, which however are not propagators of free fields. The Gaussian dominance has essentially dynamical origin, which is not clear yet, but is strongly supported by lattice calculations and hadron phenomenology. As was mentioned above the Gaussian model is quite successful in decribing NP dynamics in all situations studied heretofore. Corrections due to higher correlators in the cases studied were not large (about a few percent for the static potential).

Contrary to the QCD sum–rules MFC is applicable both at small and large distances. It includes perturbative QCD at small distances with asymp-

totic freedom, but also describes the extrapolation to large distances.

The MFC has been applied to describe the hadronic spectrum for light quarks [34], heavy quarkonia [35, 36, 37, 38, 39], heavy-light mesons [40], glueballs and hybrids [41, 42], formfactors and structure functions in the proper light–cone dynamics [43, 44], string formation [45], diffractive photoproduction [46], scattering [47, 48, 49, 50, 51, 52] and also outside of QCD to calculate the spectrum of electroweak theory [53].

There exist several reviews which are partly devoted to specific applications of MFC, namely, [54] treats mostly the hadron spectrum, scattering is discussed in [55, 56, 57], the problem of confinement is reviewed in [19], and QCD at nonzero temperature and deconfinement in [58], brief overview of MFC is presented in [59].

Another example of applications is the calculation of FC directly in the framework of a given field–theory, which is possible in the Abelian Higgs model and compact electrodynamics (see detailed review [20] and references therein).

We do not touch most of these applications in the present review, referring the reader to the cited literature. Instead, we concentrate here on the general and more fundamental questions, including definition of FC in the continuum and on the lattice, the renormalization, connection to OPE, low–energy theorems etc. We discuss the basic properties of NP QCD as a testing ground for the method, and show how they are described in the method of field correlators already in its simplest, Gaussian approximation.

The plan of the review is the following. Section 2 is devoted to the basic formalism of the method: definitions of field correlators, their elementary properties, factorization, nonabelian Stokes theorem. The physics of confinement in the language of field correlators is discussed in details in Section 3: cluster expansion, string formation and profile, perturbative properties, low energy theorems, confinement of the charges in higher representations of the gauge group and some other questions are reviewed. Section 4 treats the physics of heavy quarkonia by the method of field correlators. Finally, the deconfinement transition and aspects of NP physics at nonzero temperature are discussed in Section 5.

2 Basic formalism

In this Section we discuss some basic definitions and general formalism of nonlocal gauge-invariant field correlators as applied to $SU(N)$ Yang-Mills theory.

2.1 Definition of field correlators

For the most part of the review we are working in the Euclidean formalism and consider the Euclidean vacuum picture of QCD fields. From the QCD sum rules and heavy quarkonia spectrum analysis we know that the gluon vacuum is dense,

$$G_2 \equiv \frac{\alpha_s}{\pi}\langle F^a_{\mu\nu}F^a_{\mu\nu}\rangle \sim 0.012\,\mathrm{GeV}^4. \tag{2.1}$$

A dynamical characteristic of such stochastic vacuum is given by a set of gauge-invariant v.e.v., which in the nonabelian case have the form

$$\frac{1}{N_c}\langle \mathrm{Tr}\, G_{\mu_1\nu_1}(x_1,x_0)G_{\mu_2\nu_2}(x_2,x_0)...G_{\mu_n\nu_n}(x_n,x_0)\rangle = \Delta_{1,2...n} \tag{2.2}$$

where

$$G_{\mu_k\nu_k}(x_k,x_0) = \Phi(x_0,x_k)F_{\mu_k\nu_k}(x_k)\Phi(x_k,x_0) \tag{2.3}$$

and phase factors Φ are defined as follows

$$\Phi(x,y) = \mathrm{P}\exp i\int_y^x A_\mu dz_\mu \tag{2.4}$$

with the integration going along some curve, connecting the initial and the final points. The angular brackets are defined here as vacuum expectation value with the usual QCD weight, the exact form of which is given by

$$\langle \mathcal{Q}\rangle = \int d\mu(A)e^{-S(A)}\mathcal{Q}(A) \tag{2.5}$$

Here $S(A)$ is the standard Yang-Mills action

$$S(A) = \frac{1}{2g^2}\int d^4x\, \mathrm{Tr}\, F^2_{\mu\nu}$$

and the measure $d\mu(A)$ includes gauge-fixing and the corresponding Faddeev-Popov terms. The fermion determinant is taken to be unity in the quenched case considered here.

The most attractive feature of the nonlocal averages (2.2) is their gauge-invariance, as compared with the case of usual gauge field Green's functions $\langle A(x)A(y)..A(z)\rangle$. There is a serious price to pay, however — functions $\Delta_{1,2...n}$ depend on the form of the contours, entering (2.4). This dependence is to be cancelled in physical quantites, and we will discuss below how such cancellation occurs. It will be argued, in particular, that in Gaussian approximation contour dependence is a subleading effect.

The form of basic quantities (2.2) suggests to use the special class of gauge conditions, the so called generalized contour gauges [64, 65], to be discussed below. Of particular importance is the case of straight-line contours in (2.4), which corresponds to Fock-Schwinger (F-S) gauge condition [62, 63]. In this case any n–point correlator can be constructed using the invariant tensors $\varepsilon_{\mu\nu\sigma\rho}, \delta_{\mu\nu}$, vectors $(z_i-z_0)_\mu$ and several scalar functions, depending on relative coordinates $(z_i-z_0)^2, (z_i-z_j)^2$, where z_0 is the base point of the F-S gauge, which can be placed at the origin. The properties of these functions play a crucial role in what follows (see Section 3).

For the simplest nontrivial 2–point correlator with the points z_1 and z_2 connected by the straight line phase factors (i.e the point z_0 is chosen on the line, connecting z_1 and z_2) one parametrizes bilocal correlator conventionally as [16, 17, 18]:

$$\Delta^{(2)}_{\mu_1\nu_1,\mu_2\nu_2} = \frac{1}{N_c}\mathrm{Tr}\,\langle (F_{\mu_1\nu_1}(z_1)\Phi(z_1,z_2)F_{\mu_2\nu_2}(z_2)\Phi(z_2,z_1))\rangle =$$

$$= \frac{1}{2}\left(\frac{\partial}{\partial z_{\mu_1}}(z_{\mu_2}\delta_{\nu_1\nu_2}-z_{\nu_2}\delta_{\nu_1\mu_2})+\frac{\partial}{\partial z_{\nu_1}}(z_{\nu_2}\delta_{\mu_1\mu_2}-z_{\mu_2}\delta_{\mu_1\nu_2})\right) D_1(z_1-z_2)+$$
$$+ (\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2}-\delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})\, D(z_1-z_2) \qquad (2.6)$$

This representation does not contain CP-noninvariant part, proportional to $\varepsilon_{\mu_1\nu_1\mu_2\nu_2}$, which should be included in theories with nonzero θ-term.

Another useful representation comes if one considers correlators of CE and CM fields separately :

$$\frac{1}{N_c}\,\mathrm{Tr}\,\langle (E_i(x)\Phi(x,y)E_j(y)\Phi(y,x))\rangle =$$

$$= \delta_{ij} \left(D(z) + D_1(z) + [z_4]^2 \, \frac{dD_1(z)}{dz^2} \right) + z_i z_j \frac{dD_1(z)}{dz^2}$$

$$\frac{1}{N_c} \mathrm{Tr} \, \langle (B_i(x) \Phi(x,y) B_j(y) \Phi(y,x)) \rangle =$$

$$= \delta_{ij} \left(D(z) + D_1(z) + [\vec{z}]^2 \, \frac{dD_1(z)}{dz^2} \right) - z_i z_j \frac{dD_1(z)}{dz^2} \qquad (2.7)$$

where $z = x - y$. The following notation is useful [24]:

$$D_{||}(z) = D(z) + D_1(z) + z^2 \frac{dD_1(z)}{dz^2} \quad ; \quad D_{\perp}(z) = D(z) + D_1(z) \qquad (2.8)$$

The 2-point correlator of the dual field strength $\tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$ is constructed from dual functions $\tilde{D}, \tilde{D}_1$ in complete analogy with (2.6), the dual functions are related with D and D_1 according to

$$\tilde{D}(z) = D(z) + 2D_1(z) + z^2 \, \frac{dD_1(z)}{dz^2} \quad ; \quad \tilde{D}_1(z) = -D_1(z) \qquad (2.9)$$

It is immediately seen, that on the (anti)-selfdual configurations $D_1 \equiv 0$.

Let us discuss some basic properties of the functions introduced above. First, consider the case of QED [66]. The gauge field propagator in the Feynman gauge takes the form

$$\langle eA_\mu(x) eA_\nu(y) \rangle = \delta_{\mu\nu} \frac{1}{4\pi^2} \frac{e^2(z)}{z^2} \qquad (2.10)$$

where $z = x - y$ and the square of the invariant charge $e^2(z)$ (in coordinate space) is introduced. Differentiating both sides of (2.10) with respect to z one finds for $\langle F_{\mu\nu} F_{\rho\sigma} \rangle$ in QED:

$$D(z) \equiv 0 \quad ; \quad D_1(z) = -\frac{1}{\pi^2} \, \frac{d}{dz^2} \, \left(\frac{e^2}{z^2} \right) \qquad (2.11)$$

and

$$\langle e^2 F_{\mu\nu}(x) F_{\mu\nu}(y) \rangle = \frac{6}{\pi^2} \, \frac{1}{z^4} \, \left(1 - \frac{d\beta(e^2)}{de^2} \right) \cdot \beta(e^2) \qquad (2.12)$$

where

$$z^2 \frac{de^2}{dz^2} = \beta(e^2)$$

The formulas (2.11)–(2.12) are obtained in abelian theory. Notice that $D(z) \equiv 0$ to all orders in perturbation theory. The case of Yang-Mills theory is different from that mostly in two respects: there are perturbative contributions to the function $D(z)$ and, which is more important, both functions D and D_1 acquire NP parts, to be denoted $D^{pert}(z)$ and $D^{np}(z), D_1^{np}(z)$, respectively Pure perturbative contributions to D and D_1 are discussed in the Section 3, while most of the rest of the review is devoted to the NP parts, and we usually omit the subscripts *pert* and *np*.

The perturbation theory dictates the singular behaviour of D^{pert}, D_1^{pert} at the origin: $D^{pert}(z) \sim z^{-4}$, if $z^2 \to 0$. The NP part is normalized to the gluon condensate (2.1) according to:

$$24N_c \cdot (D^{np}(0) + D_1^{np}(0)) = \langle F^a_{\mu\nu} F^a_{\mu\nu} \rangle \equiv 4\pi^2\, G_2 \tag{2.13}$$

This information is to be used in fitting the lattice data on correlators, which we are now in the position to discuss. The first measurements were made in quenched $SU(2)$ gauge theory [21, 22]. The motivation was to correct the predictions of [60, 61] of the shift produced by the gluon condensate on the levels of heavy $\bar{q}q$ bound states: the introduction of a finite correlation length strongly reduces the effect on higher levels. The NP part exhibited an exponential dependence on the physical distance with a slope $T_g \approx 0.16$ Fm. The condensate was also determined with the result $G_2 = 0.012\text{GeV}^4$. Of course the energy scale is conventional, the string tension for $SU(2)$ being put equal to the physical string tension $\sigma \approx \frac{1}{2\pi}\text{GeV}^2$. What is actually measured are loops made of two plaquettes connected by the phase factors along straight lines, with different orientation of the plaquettes. A progress was made a few years later [24] when the correlators were measured for $SU(3)$ gauge theory in the quenched approximation, i.e. with no dynamical quarks. Besides the increase in the computer power available the progress was made possible by using cooling technique to study large distance correlations [23]. The configurations produced numerically by use of the correct action are filtered by this procedure eliminating short range fluctuations, but leaving at the same time intact long range correlations. The principle is the following: cooling is done by a local procedure which freezes singular links. It can be shown that the maximum distance L which is affected after n steps of cooling depends on n in a diffusion-like form : $L^2 = D \cdot n$. In measuring a correlation at distance d a plateau is eventually observed after a number of cooling steps, on which the value of the correlator is read, corrected for short distance fluctuations. Empirically the procedure requires a minimum distance of 3-4

lattice spacings to work. This makes hard to measure correlators at short distances. On the one hand the shortest distance observed (say 0.1 Fm) should correspond to 3-4 lattice spacings at least. On the other hand to avoid dominance of surface effects the lattice must be large enough to cover ~ 1.5 Fm in distance. In [25, 26] the correlators was measured for quenched $SU(3)$ from 0.1 Fm to 1 Fm in distance.

At large distances an exponential fit correctly describes the data. At short distances a divergent behavior $\sim z^{-4}$ is observed as predicted by perturbation theory. There is no theoretically rigorous way to separate these two contributions on the lattice. Different parametrizations were used in [24, 26] to fit the data and they proved to be equally good. One possibility is

$$\frac{1}{a^4}\, D^{lat}(z^2) = \frac{b}{|z|^4}\exp(-|z|/\lambda_a) + A_0\exp(-|z|/T_g)$$

$$\frac{1}{a^4}\, D_1^{lat}(z^2) = \frac{b_1}{|z|^4}\exp(-|z|/\lambda_a) + A_1\exp(-|z|/T_g) \tag{2.14}$$

The constants A_0, A_1 are related to the gluon condensate G_2 as follows (see (2.13)):

$$A_0 + A_1 = \frac{\pi^2}{18}\, G_2 \tag{2.15}$$

A fit to the lattice data gives for quenched $SU(3)$ (see also Fig.1)

$$T_g \approx 0.22\mathrm{Fm} \quad ; \quad G_2 = (0.14 \pm 0.02)\mathrm{GeV}^4, \tag{2.16}$$

i.e. a value large by an order of magnitude than the phenomenological value of [11]. Moreover it was found $|A_1/A_0| \approx 0.1$.

In [25] the measurements were done in full QCD with 4 staggered fermions, at different values of quark masses $m_q = 0.01/a$; $m_q = 0.02/a$. The results were in this case

$$\begin{array}{lll} am_q = 0.01 \;\; : & T_g = (0.34 \pm 0.02)\,\mathrm{Fm} & G_2 = (0.015 \pm 0.003)\,\mathrm{GeV}^4 \\ am_q = 0.02 \;\; : & T_g = (0.29 \pm 0.02)\,\mathrm{Fm} & G_2 = (0.031 \pm 0.005)\,\mathrm{GeV}^4 \end{array}$$

The value of the condensate can be extrapolated to the physical quark masses by the techniques of [11] giving $G_2 = (0.022 \pm 0.006)\mathrm{GeV}^4$, an order of magnitude smaller than the quenched value, and in agreement with phenomenological determinations [60, 61]. It is worth mentioned, however,

that even for full unquenched QCD it is rather small: $T_g = 0.16$ Fm for quenched $SU(2)$, 0.22 Fm for quenched $SU(3)$, and 0.34 Fm for full QCD with 4 flavours.

A detailed analysis has also been made of the correlators at finite temperature [25, 26]. The $O(4)$ invariance is lost at nonzero T and five independent form–factors exist instead of two at $T = 0$.

The main result is that magnetic correlators stay intact in the course of the phase transition, while electric correlators have a sharp drop (compare Fig.2 and Fig.3). Above T_c the electric longitudinal correlators are very small so that the numerical determination is affected by large errors.

It is of prime importance to stress, that electric condensate, defined by the electric parts of (2.13) does not play by itself the role of an order parameter for the confinement–deconfinement transition and, in particular, stays nonzero above T_c (since NP part of D_1 is nonzero). It is only the function D, which vanishes at the point of the phase transition and this in its turn indicates zero string tension (see 3.7) and hence deconfinement. This behaviour of correlators at the deconfinement transition is fully consistent with theoretical expectations. More on the field correlators at nonzero temperature and deconfinement transition can be found in section 5.

2.2 Relations between correlators

Correlators (2.2) with different number of F's are not completely independent quantities. The dynamical (Schwinger-Dyson type) equations for them are discussed in [70]. In this section we are concentrating on the identities which must be satisfied due to the Bianchi identities, $D_\mu \tilde{F}_{\mu\nu} = 0$, which hold true for $F_{\mu\nu}$. The fields are nonabelian and therefore the derivative of the correlator with respect to z_σ contains also the differentiation of the contour (see references [71]-[80]):

$$\frac{\partial \Phi(z,z')}{\partial z'_\gamma} = i\Phi(z,z')A_\gamma(z') + i(z'-z)_\rho \tilde{I}_{\rho\gamma}(z,z'), \tag{2.17}$$

where the following notation is used:

$$\tilde{I}_{\rho\gamma}(z,z') = \int_0^1 d\alpha\ \alpha\ \Phi(z, z+\alpha(z'-z))F_{\rho\gamma}(z+\alpha(z'-z)) \cdot \\ \cdot \Phi(z+\alpha(z'-z), z') \tag{2.18}$$

One easily obtains the relation between 2–point and 3-point correlators:

$$\varepsilon_{\mu_1\nu_1\sigma\rho}\frac{dD(z)}{dz^2} = \frac{i}{4}\varepsilon_{\mu_2\nu_2\xi\rho}\Big(\langle Tr(F_{\mu_1\nu_1}(z_1)\tilde{I}_{\sigma\xi}(z_1,z_2)F_{\mu_2\nu_2}(z_2)\Phi(z_2,z_1))\rangle -$$

$$-\langle Tr(F_{\mu_1\nu_1}(z_1)\Phi(z_1,z_2)F_{\mu_2\nu_2}(z_2)I_{\sigma\xi}(z_2,z_1)\rangle\Big) \tag{2.19}$$

The formula (2.19) is exact and also holds for the perturbative parts of the correlators. Let us focus our attention on the NP parts, assuming their regular behavior at the origin. One finds [76]:

$$\left.\frac{dD(z)}{dz^2}\right|_{z=0} = \frac{1}{96N_c}\, f^{abc}\langle F^a_{\mu_1\nu_1}F^b_{\nu_1\nu_2}F^c_{\nu_2\mu_1}\rangle. \tag{2.20}$$

In a similar way by differentiating any FC one obtains two pieces: "local" one, where the field strength operator is differentiated, and "nonlocal" term, where the parallel transporters are differentiated. Taking an exterior derivative the Bianchi identity eliminates the first term, while the second ones give rise to a higher FC. Thus one gets an infinite set of relations between FC. For some more complicated relations between FC, e.g. expressing the second derivative of $D(z)$ the reader is referred to [81].

2.3 Nonabelian Stokes theorem

The important role in the discussed formalism is played by the nonabelian Stokes theorem. There exist many derivations and interpretations of this result, see [71] - [80]. One of the simplest ways [80] to derive the Stokes theorem for the nonabelian fields is to use the so called generalized contour gauge [64] (see also [82]), where the vector-potential satisfies the following condition $t_\mu(x)A_\mu(x) = 0$ and is expressed as

$$A_\mu(x) = \int_0^1 ds \frac{\partial z_\rho(s,x)}{\partial s}\frac{\partial z_\sigma(s.x)}{\partial x_\mu}F_{\rho\sigma}(z(s,x)) \tag{2.21}$$

Here the set of contours $z_\mu = z_\mu(s,x)$ is such that

$$z_\mu(0,x) = x^0_\mu \;\; ; \;\; z_\mu(1,x) = x_\mu \;\; ; \;\; z_\mu(s,x) = z_\mu(s'', z(s',x)) \tag{2.22}$$

and $t_\mu(x) = \left.\frac{\partial z_\mu(s,x)}{\partial s}\right|_{s=1}$.

Consider now the gauge-invariant object — the Wegner–Wilson loop [83, 84] (which we denote as W–loop throughout this review):

$$W(C) = \frac{1}{N_c}\mathrm{Tr}\ \mathrm{P}\exp\left(i\oint_C A_\mu(x)dx_\mu\right) \tag{2.23}$$

where the integration goes over the closed contour C. Insertion of (2.21) into (2.23) immediately yields the nonabelian Stokes theorem for $W(C)$,

$$W(C) = \frac{1}{N_c}\mathrm{Tr}\ \mathcal{P}\exp\left(i\int_S d\sigma_{\nu\mu}(u)G_{\nu\mu}(u,x_0)\right) \tag{2.24}$$

where $G_{\mu\nu}$ is related to gauge field strength $F_{\mu\nu}$ according to (2.3) (notice, that phase factors along the contours given by $z(s,x)$ equal to unity in the gauge (2.21)). The surface ordering $\mathcal{P}$ in (2.24) is induced by linear ordering P in (2.23). The reference point x_0 is chosen arbitrarily on the surface S (for details see [73] - [80]). Contrary to the abelian case, the surface S in (2.24) must have the topology of the disk. The inclusion of higher genera is nontrivial and requires additional holonomies around fundamental cycles [77, 78, 79].

The r.h.s. of (2.24) does not depend neither on the choice of surface S nor on the choice of the contours $z(s,x)$ in (2.21) if the nonabelian Bianchi identities $D_\mu\tilde{F}_{\mu\nu} = 0$ are satisfied. To proceed, one has to consider the average of the W–loop $\langle W(C)\rangle$ over all gauge field configurations. In view of (2.24) this average can be expressed through the correlators (2.3). The average $\langle..\rangle$ and ordering $\mathcal{P}$ procedures, which should be applied to $W(C)$, are nontrivially related to each other, as we discuss it in the next section.

2.4 Stochastic vacuum and factorization of field components

Next we discuss the picture of the Gaussian vacuum. In this model, as it was already explained in the introduction, all field correlators are assumed to factorize into two-point ones. Thus one approximates the complicated ensemble of fields of nonperturbative QCD by the one satisfying Gaussian distribution. We will discuss in what follows existing lattice evidences for such assumption. It is very important to mention, that the role of stochastic variables in the approach is played by parallel transported field strength variables $G_{\mu\nu}$ (2.3), not by the gauge field potentials A_μ. While in perturbation theory one

can establish one-to-one correspondence between gauge-covariant and noncovariant descriptions [15], Gaussian dominance is gauge-invariant statement, and it is not clear, for example, which property of the propagators $\langle A..A\rangle$ it corresponds to in covariant gauges.

Whereas in a process with commuting stochastic variables the Gaussian factorization is uniquely determined, there are several possibilities for the case of non-commuting variables which we discuss now briefly. One way to factorize averages (2.2) is direct matrix factorization leading to vanishing of higher van Kampen cumulants [86, 87] and direct exponentiation:

$$\langle \mathrm{Tr}\, G(1)...G(2n)\rangle_K \sim (\langle \mathrm{Tr} G(1)G(2)\rangle\langle \mathrm{Tr} G(3)G(4)\rangle \dots \langle \mathrm{Tr} G(2n-1)G(2n)\rangle +$$
$$+ \text{ all path ordered permutations}) \tag{2.25}$$

(we remind, that all correlators of the odd power vanish in Gaussian model). This scheme is however of not predictive power if we try to evaluate the expectation value of two or more W–loops with the help of the basic two-point correlator. In order to go further in this direction we have to make asumptions on the factorization of the colour compoments of the fields directly. An obvious choice is to assume the following factorization for the components:

$$\langle G^{a_1}(1)...G^{a_n}(2n)\rangle_C sim$$
$$(\delta^{a_1a_2}\,\delta^{a_3a_4}\dots\delta^{a_{n-1}a_n}\,\langle \mathrm{Tr}\, G(1)G(2)\rangle\langle \mathrm{Tr}\, G(3)G(4)\rangle \dots \langle \mathrm{Tr}\, G(2n-1)G(2n)\rangle +$$
$$+ \text{all path ordered permutations }) \tag{2.26}$$

However the difference between the matrix van Kampen factorization $\langle . \rangle_K$ and the component factorization $\langle . \rangle_C$ is not fully ordered, and hence it would give only a correction to the two-point cumulant, if we had complete path ordering. But after applying the nonabelian Stokes theorem the field strength matrices are only surface ordered and points in the correlator, which are separated by a large number in surface ordering can be physically quite close.

These contributions can destroy the surface law and give rise to a term proportional to the 3/2-power of the area. Since surface ordering is much less treated in the literature than path ordering we discuss the problem in some detail. Let $\xi(t), \quad 0 \le t \le T$ be a non-commuting stochastic variable describing a centered Gaussian process with correlator:

$$\langle \xi(t_1)\xi(t_2)\rangle = \langle \xi(t_2)\xi(t_1)\rangle = c\cdot\langle \xi^2\rangle \phi(|t_1-t_2|) \text{ with } \phi(0)=1 \tag{2.27}$$

where c is some numerical constant of the order of 1 and function $\phi(x)$ decays with correlation length a.

In the expansion of the exponential $\langle P\exp[\int_0^T \xi(t)\,dt]\rangle$ we encounter terms like

$$\langle P\exp[\int_0^T \xi(t)\,dt]\rangle = 1 + \int_0^T dt_1 \int_0^{t_1} dt_2 \langle\xi(t_1)\xi(t_2)\rangle +$$
$$+ \int_0^T dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 \,(\langle\xi(t_1)\xi(t_2)\rangle\langle\xi(t_3)\xi(t_4)\rangle +$$
$$+ \langle\xi(t_1)\xi(t_3)\rangle\langle\xi(t_2)\xi(t_4)\rangle + \langle\xi(t_1)\xi(t_4)\rangle\langle\xi(t_2)\xi(t_3)\rangle) + \ldots \qquad (2.28)$$

For the first nontrivial term we obtain:

$$\int_0^T dt_1 \int_0^{t_1} dt_2 \langle\xi(t_1)\xi(t_2)\rangle = c\langle\xi^2\rangle aT \qquad (2.29)$$

where the value of c depends on the form of the correlation function. The next terms crucially depend on the path ordering (we assume $T \gg a$):

$$\int_0^T dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 \langle\xi(t_1)\xi(t_2)\rangle\langle\xi(t_3)\xi(t_4)\rangle = \frac{1}{2}c^2\langle\xi^2\rangle^2(aT)^2 \qquad (2.30)$$

$$\int_0^T dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \int_0^{t_3} dt_4 \langle\xi(t_1)\xi(t_3)\rangle\langle\xi(t_2)\xi(t_4)\rangle = c'\langle\xi^2\rangle^2 a^3 T \qquad (2.31)$$

The term of equation (2.29) and the term in equation (2.30) contributes to the exponential

$$\exp[\frac{1}{2}c\langle\xi\rangle^2 aT]$$

whereas the term of equation (2.30) is only of order T and therefore modifies the coefficient of T in the exponent by a term of the order $c'\langle\xi^2\rangle^2 a^3$. The details of the factorization can thus always be absorbed in a modification of the value of $\langle\xi^2\rangle$.

Let us now consider a stochastic variable on a plane $\xi(u,v)$ with similar properties as the one discussed above (2.27) and investigate the surface ordered expectation value:

$$\langle\mathcal{P}\exp\int_0^R du \int_0^T dv\; \xi(u,v)\rangle,$$

where $\mathcal{P}$ stands for surface ordering analogously to (2.24). The first nontrivial term in the expansion of the exponential, analogous to (2.29) yields a contribution proportional to

$$\langle\xi^2\rangle a^2 RT$$

independent of surface ordering. But the terms containig the expectation value of four values of the stochastic variable can behave quite differently: Let again f_1, f_2, f_3, f_4 denote four ordered points on the surface. Then the integral over the ordered expression

$$\langle\xi(f_1)\xi(f_2)\rangle\langle\xi(f_3)\xi(f_4)\rangle$$

yields $\langle\xi^2\rangle^2 a^4 (RT)^2$ which together with the former term exponentiates and contributes to the area law

$$\exp[\frac{1}{2}c\langle\xi^2\rangle a^2 RT],$$

but the ordering

$$\langle\xi(f_1)\xi(f_3)\rangle\langle\xi(f_2)\xi(f_4)\rangle$$

is not simply a correction to the first term: if f_1 and f_3 are close but in different rows the points f_2, f_4 can vary over the full row, thus yielding a factor of the order T , and the final contribution will be of the order

$$\langle\xi^2\rangle^2 a^3 T(RT)$$

and thus not fit into an exponentiation. So in contrast to the path ordering where different clusterings, as long as they agreed in the leading (ordered) term exponentiated to an perimeter (area) law, in surface ordering we need a fine tuning in order to get the latter.

This is a reflection of the rather artificial feature of the Gaussian model that we have always to treat correlators where the reference point is fixed from the beginning and hence might be quite distant from the connection line of the two external points.

We can exclude this unphysical contributions if we specify the factorization prescription (2.26) by the following ordering rule [45]: if the expectation value is expanded as a power series in the two-point correlators only the leading powers of the ratio of the correlation length and the linear dimensions of the loops have to be taken into account in each order.

It is easy to see that the factorization (2.26) together with this rule leads in the case of one loop to the same area law as the matrix factorization (2.25). It will be referred to in the following as the modified component factorization in the model of the stochastic vacuum.

It is instructive to mention here a proposal of [88] based on a domain picture of the QCD vacuum in which these problems do not occur and the formal modification of the component factorization is justified from a physical point of view.

3 Confinement and field correlators

In this chapter we discuss the following question: how confinement is related with the properties of FC, what is the physical mechanism behind it and how the cluster expansion operates in the quantitative description of confinement. In what follows the role of the order parameter of confinement is played by the averaged W–loop [83, 84] (we assume quenched approximation)

$$\langle W(C)\rangle = \frac{1}{N_c}\langle \mathrm{Tr}\ \mathrm{P}\exp(i\oint_C A_\mu dx_\mu)\rangle = \exp[-V(R)T] \tag{3.1}$$

This is the true order parameter only in case when light quarks are absent and $V(R)$ is the interquark potential for (infinitely) heavy static quarks. Confinement for light quarks and gluons is much more intricate phenomenon, which is, however, ensured by the same FC as for heavy quarks (see [34]).

This Section has the following structure: confinement in terms of the cluster expansion is explained in subsection 3.1, corresponding physical aspects and confinement of the charges in higher representations are reviewed in 3.2, abelian projection method in terms of field correlators briefly commented in subsection 3.3, perturbative structure of the cluster expansion is explained in subsection 3.4, the string profile is discussed in subsection 3.5, and, finally, subsection 3.6 is devoted to the explanation of the model consistency with the low–energy theorems and related questions.

3.1 Confinement and cluster expansion

The nonabelian Stokes theorem and cluster expansion theorem [86] applied to the W–loop yields the following result:

$$\langle W(C)\rangle = \frac{1}{N_c}\left\langle \mathrm{Tr}\ \mathrm{P}\exp(i\oint_C A_\mu(x)dx_\mu)\right\rangle =$$

$$= \frac{1}{N_c}\left\langle \mathrm{Tr}\ \mathcal{P}\exp(i\int_S G_{\mu\nu}(u,x_0)d\sigma_{\mu\nu}(u))\right\rangle =$$

$$= \frac{1}{N_c}\ \mathrm{Tr}\ \ \exp\left(\sum_{n=1}^{\infty}\frac{i^n}{n!}\int d\sigma(1)..d\sigma(n)\langle\langle G(1)..G(n)\rangle\rangle\right) \tag{3.2}$$

where we have suppressed the indices, $d\sigma(k)G(k) = G_{\mu\nu}(u_k,x_0)d\sigma_{\mu\nu}(u_k)$, and we have used irreducible cumulants. Due to colour neutrality of the vacuum

all cumulants are proportional to the unit matrix in the colour space which makes unnecessary colour ordering in the r.h.s. of (3.2).

The irreducible correlators $\langle\langle ..\rangle\rangle$ are straightforwardly extracted from (3.2) comparing terms of the same order in G. Notice, that there is only one ordering in the expression (3.2) and this ordering is induced by the P–ordering in the original loop (3.1).

The most important for us is the bilocal correlator, which is defined as

$$\langle G_{\mu_1\nu_1}(x_1)G_{\mu_2\nu_2}(x_2)\rangle = \langle\langle G_{\mu_1\nu_1}(x_1)G_{\mu_2\nu_2}(x_2)\rangle\rangle + \langle G_{\mu_1\nu_1}(x_1)\rangle\langle G_{\mu_2\nu_2}(x_2)\rangle$$

Since $\langle G(k)\rangle \equiv 0$, one can rewrite (3.2) identically as

$$\langle W(C)\rangle = \frac{1}{N_c} \mathrm{Tr}\ \exp\left(-\frac{1}{2}\int\limits_S\int\limits_S d\sigma_{\mu\nu}(u)d\sigma_{\rho\sigma}(v)\Lambda_{\mu\nu,\rho\sigma}(u,v,S)\right) \qquad (3.3)$$

where we have defined the global correlator $\Lambda(u,v,S)$ [89]

$$-\frac{1}{2}\,\Lambda_{\mu\nu,\rho\sigma}(u,v,S) \equiv -\frac{1}{2}\,\langle\langle G_{\mu\nu}(u,x_0)G_{\rho\sigma}(v,x_0)\rangle\rangle +$$
$$+\sum_{n=3}^{\infty}\frac{i^n}{n!}\int d\sigma(3)..d\sigma(n)\Big[\langle\langle G_{\mu\nu}(u,x_0)G_{\rho\sigma}(v,x_0)G(3)..G(n)\rangle\rangle +$$
$$+\ \mathrm{perm}(1,2,..n)\Big] \qquad (3.4)$$

and $\mathrm{perm}(1,2,..n)$ stands for the sum of terms with different ordering of $G(u,x_0)=G(1)$ and $G(v,x_0)=G(2)$ with respect to all other factors $G(k)$. Since $W(C)$ does not depend on the shape of the surface S, the dependence of the global correlator on its arguments is such, that r.h.s. of (3.3) is independent of the choice of S but depends on the contour C. This circumstance explains the termin "global correlator" used for $\Lambda(u,v,S)$ in contrast to local correlators which enter in the r.h.s. of (3.4) (note however, the specific meaning of the definition "local" – correlators $\langle\langle G(1)..G(k)\rangle\rangle$ depend on the points $z_1,..,z_k$ as well as on the paths, entering via transporters $\Phi(z,x_0)$).

If one integrates over all but one variables in (3.3), it leads to the effective integrand

$$Q_{\mu\nu}(u,S) \equiv \frac{1}{2}\int\limits_S d\sigma_{\rho\sigma}(v)\Lambda_{\mu\nu,\rho\sigma}(u,v,S) \qquad (3.5)$$

In the confining phase one expects for large contours C the minimal area law of W–loop, which implies that $Q_{\mu\nu}$ in this limit does not depend on the point

u when S is the minimal area surface and simply coincides with the string tension σ, while for the arbitrary surface one can identify $Q_{\mu\nu}$ as

$$Q_{\mu\nu}(u, S) = P_{\mu\nu} \cdot \sigma \tag{3.6}$$

where $P_{\mu\nu}$ projects onto the minimal surface. Conversely if $Q_{\mu\nu}$ does not have constant limit for large S then the area law of W–loop does not hold.

To calculate $Q_{\mu\nu}(u, S)$ one can for simplicity take x_0 in (3.4) to coincide with u. Then the lowest order FC in (3.4) depends only on two points (as in (2.6)). In what follows we concentrate on contributions to σ and therefore take for simplicity a planar contour C with the minimal surface S lying in the plane.

Insertion of (2.6) into (3.4) yields the area law of W–loop with the string tension σ

$$\langle W(C)\rangle = \exp(-\sigma S_{min}),$$

$$\sigma^{(2)} = \frac{1}{2}\int d^2x \, D(x) \tag{3.7}$$

Note that D_1 does not enter σ, but gives rise to the perimeter term. It can be clearly seen from the definition (2.6) since D_1 by definition contains derivatives. One concludes therefore, that the presence of nontrivial non-perturbative contribution to the function $D(x)$ such, that (3.7) is nonzero but finite implies confinement in Gaussian approximation. The lattice data described above support this since $D(x)$ demonstrates exponential falloff at large distances and hence

$$\sigma \sim G_2 \cdot T_g^2$$

Till now we have discussed confinement in terms of the lowest cumulant – $D(x)$, which is justified when stochasticity condition (3.9) is fulfilled and $D(x)$ gives a dominant contribution. Let us now turn to other terms in the cluster expansion (3.2). It is clear that the general structure of higher cumulants is much more complicated than (2.6), but there always Kronecker-type terms $D(x_1, x_2, ..., x_n) \prod \delta_{\mu_l \mu_k}$ present similar to $D(x-y)$ in (2.6) and other terms containing derivatives like $D_1(x-y)$. The term $D(x_1, x_2, ..., x_n)$ contributes to string tension. This means that the string tension in the general case is a sum,

$$\sigma = \sum_{n=2}^{\infty} \sigma^{(n)} \ , \ \sigma^{(n)} \sim \int \langle\langle G(1)G(2)...G(n)\rangle\rangle d\sigma(2)...d\sigma(n) \tag{3.8}$$

If one assumes, that all gluon (irreducible) condensates and the corresponding correlation lengths have the same order of magnitude, then the expansion in (3.8) is in powers of $(\lambda\sqrt{G_2}T_g^2)$, and if this parameter is small,

$$\lambda\sqrt{G_2}T_g^2 \ll 1, \tag{3.9}$$

one gets the limit of Gaussian stochastic ensemble where the lowest (quadratic in $G_{\mu\nu}$) cumulant is dominant. The value of $\sqrt{G_2}$ in (3.9) is a typical scale of vacuum fields, which can be estimated from the gluon condensate, $\sqrt{G_2} \sim (0.6\,\mathrm{GeV})^2$, while $\lambda \sim 1$ takes into account possible cancellations in the magnitude of the cumulant, and in this way characterizes the deviation of the vacuum ensemble from Gaussian one. Taking into account, that $\sqrt{G_2}$ in $W(C)$ enters on a plane with fixed (ij), i.e. $\sqrt{G_2} \sim \sqrt{\langle G_{ij}^2\rangle}$, while the gluonic condensate is a sum $\sum_{ij}\langle G_{ij}^2\rangle$, and the numerical value of T_g from lattice calculations $T_g \sim 0.2-0.3$ fm, one obtains as the parameter of cluster expansion $\lambda G_2 T_g^2 \sim \lambda \cdot 0.1$.

It is of prime importance to stress, that the relative contribution of different terms in the r.h.s. of the expression (3.8) depends on the chosen integration surface. Since the l.h.s. of (3.8) does not depend on the surface, the lowest cumulant dominance is expected to hold for the minimal surface in Gaussian dominance scenario, while contributions from higher terms are to become important for arbitary nonminimal surface in order to cancel artificial surface dependence. In can be said therefore, that the actual value of the string tension is determined by the total sum (3.8) in any case: the stochastic one (actually governed by Gaussian average for the minimal surface) and coherent one (when all terms are of the same order even for the minimal surface).

Summarizing, when the stochasticity condition (3.9) holds, the lowest term, $\sigma^{(2)}$, dominates in the sum; in the general case all terms in the sum (3.8) might be important. Therefore the smaller the correlation length T_g is, the better Gaussian model of confinement works. It is worth mentioning, that this limit is opposite to the sum rule limit (sometimes called Leutwyler-Voloshin limit), where the condensate is assumed to be constant in the position space, i.e. using the language of MFC $T_g \to \infty$. From this point of view the analogy between Gaussian dominance in the stochastic picture and "vacuum insertions" procedure in OPE is not straightforward.

3.2 Physical mechanism of confinement

The described formalism is valid both for abelian and nonabelian theories, and it is interesting whether the area law and nonzero string tension could be valid also for QED (or $U(1)$ in the lattice version [90]). To check it let us apply the operator $\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\frac{\partial}{\partial x_\alpha}$ to both sides of (2.6) [76].

In the abelian case, when Φ cancel in (2.2) and $F = G$, one obtains

$$\partial_\alpha \langle \tilde{F}_{\alpha\beta}(x) F_{\lambda\sigma}(y)\rangle = \varepsilon_{\lambda\sigma\gamma\beta}\partial_\gamma D(x-y), \tag{3.10}$$

where $\tilde{F}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}F_{\mu\nu}$.

If magnetic monopoles are present in abelian theory (e.g. external Dirac monopoles) with the current $\tilde{j}_\mu$, one has

$$\partial_\alpha \tilde{F}_{\alpha\beta}(x) = \tilde{j}_\beta(x) \tag{3.11}$$

In absence of magnetic monopoles (for pure QED) the abelian Bianchi identity requires that

$$\partial_\alpha \tilde{F}_{\alpha\beta} \equiv 0 \tag{3.12}$$

Thus for QED (without magnetic monopoles) the function $D(x)$ vanishes due to (3.11) and hence confinement is absent (compare with (2.11).

In the compact version of $U(1)$ magnetic monopoles are present and the lattice formulation of the method would predict the confinement regime with nonzero string tension [91]. The latter can be connected through $D(x)$ to the correlator of magnetic monopole currents. Indeed, multiplying both sides of (3.10) with $\frac{1}{2}\varepsilon_{\lambda\sigma\gamma\delta}\frac{\partial}{\partial y_\gamma}$ one obtains

$$\langle \tilde{j}_\beta(x)\tilde{j}_\delta(y)\rangle = \left(\frac{\partial}{\partial x_\alpha}\cdot\frac{\partial}{\partial y_\alpha}\delta_{\beta\delta} - \frac{\partial}{\partial x_\beta}\frac{\partial}{\partial y_\delta}\right) D(x-y) \tag{3.13}$$

This identically satisfies monopole current conservation: applying $\frac{\partial}{\partial x_\beta}$ or $\frac{\partial}{\partial y_\delta}$ to both sides of (3.13) gives zero. More on MFC in abelian theories can be found in [20].

Let us turn now to the nonabelian case, again assuming stochasticity condition (3.9). Applying as in the Abelian case the operator $\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\frac{\partial}{\partial x_\alpha}$ to the r.h.s. of (2.6), one obtains the relation given above in (2.19) Especially simple form of (2.19) occurs when tending x to y; one obtains equation (2.20), which can be rewritten as

$$\left.\frac{dD(z)}{dz^2}\right|_{z=0} \sim \langle E_i^a E_j^b B_k^c\rangle \varepsilon_{ijk} f^{abc} \tag{3.14}$$

Thus confinement (nonzero σ due to nonzero D) is related in nonabelian case with the presence of purely nonabelian correlator.

To see the physical meaning of this correlator one can visualize magnetic and electric field strength lines (FSL) in the space. Each magnetic monopole is a source of FSL, whether it is a real object (classical solution or external object like Dirac monopole) or lattice artefact. In nonabelian theory these lines may form branches and e.g. electric FSL may emit a magnetic FSL at some point, playing the role of magnetic monopole at this point. This is what exactly nonzero triple correlator $\langle \mathrm{Tr}\, FFF \rangle$ implies. Note that this could be a purely quantum effect and no real magnetic monopoles (as classical solutions or lattice artefacts) are necessary for this mechanism of confinement.

The formula (3.14) demonstrates, that in strict Gaussian approximation $D(z^2)' = 0$, since the r.h.s. is proportional to the condensate of the third power. The extended Gaussian approximation, which keeps only 2– and 3–point correlators allows to have nonzero $D(z^2)' = 0$. It does not automatically mean, however, that the contribution of the triple (and higher odd order) condensates themselves to the physical quantities (like string tension) is comparable with that of 2–point one (see (3.8) and discussion below (3.9)). It should also be noticed, that despite the first derivative of the function $D(z)$ vanishes at the origin in Gaussian vacuum, the second and higher ones of even order do not (see [81]).

We conclude this section with discussion of confinement for charges in higher representations. Most of the models of NP QCD are designed to describe confinement of colour charge and anticharge in the fundamental representation of the gauge group $SU(3)$, i.e. the area law for the simplest W–loop and hence linear potential between static quark and antiquark. At the same time Gaussian dominance scenario naturally describes lattice data on the interaction between static charges in higher $SU(3)$ representations.

We define static potential between sources at the distance R in the given representation D as:

$$V_D(R) = -\lim_{T\to\infty} \frac{1}{T} \log \langle W_D(C) \rangle, \tag{3.15}$$

where the W–loop $W_D(C)$ for the rectangular contour $C = R \times T$ in the "34" plane is given by

$$\langle W_D(C) \rangle = \left\langle \mathrm{Tr}_{\,D}\, \mathrm{P} \exp\left(i \int\limits_C A^a_\mu T^a dz_\mu \right) \right\rangle \tag{3.16}$$

The $SU(3)$ representations $D = 3, 8, 6, 15a, 10, 27, 24, 15s$ are characterized by $3^2 - 1 = 8$ hermitian generators T^a which satisfy the commutation relations $[T^a, T^b] = i f^{abc} T^c$. One of the main characteristics of the representation is an eigenvalue of quadratic Casimir operator $\mathcal{C}_D^{(2)}$, which is defined according to $\mathcal{C}_D^{(2)} = T^a T^a = C_D \cdot \hat{1}$. Following the notations from [92] we introduce the Casimir ratio $d_D = C_D/C_F$, where the fundamental Casimir $C_F = (N_c^2 - 1)/2N_c$ equals to $4/3$ for $SU(3)$. The invariant trace is given by $\mathrm{Tr}_D \hat{1} = 1$.

The potential (3.15) with the definition (3.16) admits the following decomposition

$$V_D(R) = d_D V^{(2)}(R) + d_D^2 V^{(4)}(R) + ..., \tag{3.17}$$

where the part denoted by dots contains terms, proportional to the higher powers of the quadratic Casimir as well as to higher Casimirs. It is worth mentioned, that the term $V^{(2)}(R)$ is a sum of the bilocal correlator contribution and linear Casimir contributions coming from higher correlators.

The fundamental static potential contains perturbative Coulomb part, confining linear and constant terms

$$V_D(R) = \sigma_D R - v_D - \frac{e_D}{R} \tag{3.18}$$

The Coulomb part is now known up to two loops [95, 96] and is proportional to C_D. The "Casimir scaling hypothesis" [97], see also [98, 99, 100, 101] declares, that the confinement potential is also proportional to the first power of the quadratic Casimir C_D, i.e. all terms in the r.h.s. of (3.17) are much smaller than the first one. In particular, for the string tensions one should get $\sigma_D/\sigma_F = d_D$.

This scaling law is in perfect agreement with the results found in [92] (see also [102]). It is easy to see that Gaussian cumulant in (3.16) is expressed through C_D and representation–independent averages as

$$\mathrm{Tr}_D \langle F(1)F(2)\rangle = \frac{C_D}{N_c^2 - 1} \langle F^a(1)F^a(2)\rangle = \frac{d_D}{2N_c} \langle F^a(1)F^a(2)\rangle, \tag{3.19}$$

so Gaussian approximation satisfies "Casimir scaling law" exactly. This fact does not depend on the actual profile of the potential. It could happen, that the linear potential observed in [92] is just some kind of intermediate distance characteristics and changes the profile at larger R (as it actually should happen due to the screening of the static sources by dynamical gluons

from the vacuum) The coordinate dependence of the potential is not directly related to the Casimir scaling, and can be analized at the distances which are small enough not to be affected by the screening effects (see discussion in [104, 105, 106]).

The quantitative analysis [105] of the data from [92] shows, that scaling holds with an accuracy of a few percents. The Fig. 4, taken from the paper [92] shows the potential between static sources in the different representations of the gauge group $SU(3)$. Lines on this figure correspond to the fit of the fundamental static potential multiplied by the corresponding ratio of the Casimir factors. The picture demonstrates nice agreement with the Casimir scaling hypothesis. It is worth mentioning that most of the microscopic models used in NP QCD (e.g. instanton model, center vortex model, abelian dominance hypothesis) encounter difficulties in explanation of Casimir scaling [104, 106].

In the MVC the Casimir scaling has two important features. First, it is the direct consequence of the Gaussian dominance hypothesis since bilocal correlator provides the exact Casimir scaling. On the other hand, it implies the cancellations of C_D^2 – proportional terms and higher ones in the cluster expansion of (3.16). Physically, it means the picture of the vacuum, made of relatively small colour dipoles with weak interactions between them. One can imagine two possible scenario. According to the first one, Casimir scaling is just a consequence of Gaussian dominance. It this case any higher cumulant contributes to physical quantities much less than the Gaussian one due to dynamical reasons. There is also the second possibility, when each higher term in the expansion (3.16) is not small, but their sum demonstrates strong cancellations of Casimir scaling violating terms. These pictures are in close correspondence to the stochastic versus coherent vacuum scenario, discussed above. The clarification of the situation in future lattice measurements is important to understand the NP structure of QCD vacuum in gauge-invariant terms.

3.3 Perturbative analysis of correlators in gluodynamics

Perturbative expansion of FC has specific features due to the presence of parallel transporters in the definition of FC, which should also be expanded and generate new class of diagrams. In this respect FC are similar to W–

loops. At the same time since FC contains a product of operators one expects at small distances singular contributions as in the OPE coefficient functions. Let us briefly describe the renormalization procedure for W–loops. It was studied both for smooth and cusped contours [112, 113]. In the first case the result is:

$$W(C) = Z \cdot W_{ren}(C) \tag{3.20}$$

where the (divergent) Z-factor contains linear divergencies arising from the integrations over the contour

$$Z \sim \exp\left(c \int\limits_C dx_\mu \int\limits_C dy_\nu D_{\mu\nu}(x-y)\right) \sim \exp\left(c\,\frac{L}{a}\right) \tag{3.21}$$

where L is the length of the contour, a – ultraviolet cutoff and c – numerical constant not important for us here. All logarithmic divergencies are absorbed into the renormalized charge $g_{ren}(\mu)$ defined at the corresponding dynamical scale μ. It is worth noting that a single diagram in the perturbative expansion of the r.h.s. of (3.20) typically gives contributions to both Z-factor and $W_{ren}(C)$, i.e. contain linearly and logarithmically divergent parts. Hence the factorization in the r.h.s. of (3.20) has double-faced nature - it is factorization of the diagrams together with the separation of contributions, coming from dangerous integration regions (i.e. linearly divergent terms) for each diagram itself. As a result contour divergencies can be separated as a common Z-factor, which physically renormalizes the bare mass of the test particle, moving along the contour, which forms W–loop. Therefore the W–loop can be calculated or measured on the lattice only modulo divergent Z-factor. However if one extracts the physical quantities from the W–loop average (e.g. by forming the Creutz ratio) then only logarithmic charge renormalization causes an observable effect.

If the contour possesses cusps (and selfintersections) situation becomes more complicated. Namely, each cusp leads to its own local Z – factor, depending on the cusp angle (but not on other characteristics of the contour). The expression (3.20) for the loop with k cusps having angles γ_i is to be modified in the following way [112, 113]:

$$W(C) = \prod_{i=1}^{k} Z_i\left(\gamma_i\right) \cdot W_{ren}(C) \tag{3.22}$$

The mixing between linear and logarithmic divergencies, mentioned above has some specific features for the loops with cusps. Namely, each cusp in

addition to the linearly divergent factors, as in the smooth contour case introduces logarithmically divergent terms which have nothing to do with the coupling constant renormalization and can be subtracted by their own $Z(\gamma_i)$–factors. There are two types of terms of that kind, those, depending on the cusp angle and others which do not depend on it. The latter play important role in the perturbative expansion of FC. To see this, note, that FC can be easily obtained from the W–loop functional. Indeed, differentiating (3.2) with respect to $\delta\sigma_{\mu\nu}(x_i)$ and assuming that the contour C connects the points u, v with the reference point x_0 along the straight lines, one finds for the bilocal correlator:

$$\frac{1}{N_c}\mathrm{Tr}\,\langle\langle G_{\mu\nu}(u,x_0)G_{\rho\sigma}(v,x_0)\rangle\rangle = \frac{\delta^2\langle W(\tilde{C})\rangle}{\delta\sigma_{\mu\nu}(u)\delta\sigma_{\rho\sigma}(v)} \tag{3.23}$$

and analogous expressions for higher cumulants. Note, that this contour generally has at least two cusps (typically even four, if the reference point x_0 does not coincide with one of the correlator arguments).

Comparing (3.22) and (3.23) one concludes, that the derivatives in the r.h.s. of (3.23) act on the renormalized part of the W–loop as well as on the Z – factors. As a result a mixing between different terms appears and contour divergencies cannot be separated in a simple way in the case of FC, and the perturbative behaviour of FC is controlled by both ultraviolet and contour logarithmic divergencies.

Direct perturbative analysis of the simplest nontrivial bilocal FC [114, 115] shows, that this is indeed the case. In a particular case where the reference point coincides with one of correlator arguments the perturbative expressions for the functions D and D_1 at the next-to-leading order reads [114]:

$$D^{(1)}(z^2) = g^2\frac{N_c^2-1}{2}\frac{1}{\pi^2 z^4}\frac{\alpha_s}{\pi}\left[-\frac{1}{4}L+\frac{3}{8}\right] \tag{3.24}$$

$$D_1^{(1)}(z^2) = g^2\frac{N_c^2-1}{2}\frac{1}{\pi^2 z^4}\frac{\alpha_s}{\pi}\cdot\left[\left(\frac{\beta_1}{2N_c}-\frac{1}{4}\right)L+\frac{\beta_1}{3N_c}+\frac{29}{24}+\frac{\pi^2}{3}\right] \tag{3.25}$$

with $L=\log(e^{2\gamma}\mu^2 z^2/4)$, μ being a renormalization scale in the $\overline{MS}$ scheme and the first coefficient of the β-function $\beta_1=(11N_c-2f)/6$.

Note, that at the tree level D-function receives zero contribution while D_1 is:

$$D_1^{(0)}(z^2) = \frac{N_c^2-1}{2N_c}\,\frac{g^2}{\pi^2 z^4} = \frac{16}{3}\,\frac{\alpha_s}{\pi}\,\frac{1}{z^4} \tag{3.26}$$

Since the string tension is an integral over $D(x)$ (3.24) tells that the $\mathcal{O}(g^4)$ perturbative contribution to $D(x)$ produces nonzero string tension, which physically has no sense and should be cancelled by other terms of the cluster expansion. Such cancellation was indeed shown to occur [89] in perturbation theory, so string tension is exactly zero at any finite order of the perturbation theory in the gauge–invariant formalism considered, i.e. if the latter is developed in terms of FC.

3.4 The string profile

It was already mentioned above that the main phenomenon behind the physics of confinement is the formation of the string between the colored sources. It is this string which mediates attraction between quark and antiquark with the force of roughly 15 tons.

An immediate question arises: what does the string consist of? Or in other words; what is the distribution of fields inside the string?

Before answering this question, two specifications are needed. First, speaking about field in the string, one should always compare it to the field in the vacuum, since only this difference makes the string. Second, in the QCD vacuum, which is stochastic by necessity as discussed above, the gauge invariant field distribution can be measured only with the help of field correlators.

It is instructive to distinguish two general types of correlators in this respect:

i) connected correlators – correlators of colored objects, which we discussed above, where the trace operation appears only once (if at all)

$$\langle\langle \mathrm{Tr}\, G_{\mu\nu}(u, x_0) G_{\rho\sigma}(v, x_0)\rangle\rangle$$

ii) disconnected correlators – correlators of two or more white objects, like

$$\langle\langle \mathrm{Tr}\, F^2_{\mu\nu}(x)\, \mathrm{Tr}\, F^2_{\lambda\gamma}(y)\rangle\rangle$$

In nonabelian theory, unlike in QED, those two correlators correspond to two very different probes, which can be made with fields in the string. In the first case one measures the color field distribution as it is, in the second one measures the "interaction" between two white objects: the string and the neutral plaquette, which can exchange only white objects – glueballs in case of gluodynamics. These approaches are complementary to each other and in

both cases one gets some gauge-invariant characteristics of the problem. In what follows we will compare the results of analysis performed with these two kinds of probes.

We start with the case of the connected correlators. Using Monte-Carlo (MC) technique, in [27, 28, 29] the study was made of the spatial distribution of the components of the field strength tensor in presence of a static $q\bar{q}$ pair. Following [28], we define:

$$\rho_{\mu\nu} = \frac{\langle \mathrm{Tr}\,(W \cdot \Phi \cdot P_{\mu\nu}(x_\parallel, x_\perp)\Phi^+)\rangle}{\langle \mathrm{Tr}\ W\rangle} - 1 \tag{3.27}$$

where W is a W–loop, Φ is a phase factor and $P_{\mu\nu}$ is the plaquette, oriented in order to give the desired component of the field.[1] The coordinates $x_\parallel, x_\perp$ measure the distance from the edge of the W–loop and from the plane defined by the loop, respectively. In the naive continuum limit $a \to 0$,

$$\rho_{\mu\nu} \simeq a^2 \langle F_{\mu\nu}\rangle_{q\bar{q}} \tag{3.28}$$

where $\langle F_{q\bar{q}}\rangle$ is the average value of the complicated Wilson contour with plaquette $P_{\mu\nu}$ reduced to $F_{\mu\nu}a^2$. In [29] a 16^4 lattice was used, taking a 8×8 W–loop and $\beta = 2.50$, which is inside the scaling window for the fields. Moving the plaquette in and outside the plane defined by the W–loop, one obtains a map of the spatial structure of the field as a function of $x_\parallel$ and $x_\perp$. Using a controlled cooling technique, one eliminates the short-range fluctuations [23]. The long–range non–perturbative effects survive longer to the cooling procedure, showing a plateau of 10–14 cooling steps, while the error becomes smaller. A similar behaviour has been observed for the string tension and the magnitude of string tension at the plateau coincides with the full uncooled value to a good approximation, which confirms the physical significance of string measurements, done in [27, 28, 29].

The cooling technique allows to disentangle the signal from the quantum noise with a relatively small statistics. The general patterns of the field configurations are briefly resumed in the following figures, while the complete results for the field–strength tensor are contained in Tables of [28, 29]. Fig. 5 represent a map of the spatial behaviour of the longitudinal component of the CE field. From Figs. 5, it is seen that the parallel electric field is squeezed in flux tube.

[1] Indices μ, ν in (3.27) are not usual tensor indices but numerical arguments, informing about the chosen direction of the plaquette.

We denote direction along the $q\bar{q}$ axis $x_\parallel = x_1$, while that of $x_\perp = x_2$, and the Euclidean temporal axis is x_4. All the construction is taken at a fixed value of x_3.

Using the non-abelian Stokes theorem and the cluster expansion (see chapter 3) for $\rho_{\mu\nu}$ one has

$$\rho_{\mu\nu}(x_1, x_2, x_4) = a^2 \int d\sigma_{14}(x_1', x_4') \Lambda_{\mu\nu} \tag{3.29}$$

where

$$\Lambda_{\mu\nu} = \frac{1}{N_c} \mathrm{Tr} \, \langle E_1(x_1', 0, x_4') \Phi F_{\mu\nu}(x_1, x_2, x_4) \Phi^+ \rangle + ..., \tag{3.30}$$

Φ is the parallel transporter from the point $(x_1', 0, x_4')$ to (x_1, x_2, x_4), and dots imply contribution of higher order cumulants containing additional powers of E_1.

We shall keep throughout this section only the lowest cumulants (containing lowest power of E_1) and compare our prediction with the MC data of previous sections. The bilocal correlator $\Lambda_{\mu\nu}$ can be expressed in terms of two independent Lorentz scalar functions $D((x_\mu - x_\mu')^2)$, $D_1((x_\mu - x_\mu')^2)$ (see chapter 2)

$$\Lambda_{14} = D + D_1 + (h_1^2 + h_4^2) \frac{dD_1}{dh^2} \tag{3.31}$$

$$\Lambda_{24} = (h_1 h_2) \frac{dD_1}{dh^2} \quad , \quad \Lambda_{34} = (h_1 h_3) \frac{dD_1}{dh^2} \tag{3.32}$$

$$\Lambda_{23} \equiv 0, \quad \Lambda_{13} = (h_3 h_4) \frac{dD_1}{dh^2} \; ; \; \Lambda_{12} = (h_2 h_4) \frac{dD_1}{dh^2} \tag{3.33}$$

Here $h_\mu = (x - x')_\mu$.
Since all construction in Fig. 5 is at $x_3 = x_3' = 0$ we have $h_3 \equiv 0$ and hence

$$\rho_{23} = \rho_{34}^c = \rho_{13}^c \equiv 0 \tag{3.34}$$

The only nonzero components are $\Lambda_{14}, \Lambda_{24}$ and Λ_{12}.

When $x_4 = 0$ (and this is where measurements of ρ_{12}^c have been done), ρ_{12} vanishes because of antisymmetry of Λ_{12} in h_4. Hence only ρ_{14} and ρ_{24} are nonzero, and only those have been measured to be nonzero.

On the Fig.5 (a,b,c) the field distributions $(\rho_{14}(x_1, x_2))^2$ are shown for different quark separations, $R = T_g, 5T_g$ and $15T_g$. The string profile, i.e. $(\rho_{14}(x_2))^2$ distribution in the middle of the string, for these quark separations is shown in Fig.5 (d). One can see a clear string-like formation with the width

of $2.2T_g$. The mean value of the CE field in the middle of string does not depend on R for R greater then $5T_g$.

To make comparison with data more quantitative, one can exploit the exponential form of D, D_1 [24]

$$D_1(h^2) = D_1(0)\ \exp(-\delta_1|h|);\ \ D(h^2) = D(0)\ \exp(-\delta|h|) \tag{3.35}$$

$$D_1(0) \approx \frac{1}{3}D(0);\quad \delta_1 \approx \delta$$

Inserting this into (3.29), we have

$$\rho_{14}(x_1,x_2;0) = a^2\int_0^R dx_1' \int\limits_{-\frac{T}{2}}^{\frac{T}{2}} dx_4' \left[D(0)+D_1(0)-\frac{(h_4^2+h_1^2)}{2h}D_1(0)\right]e^{-\delta h} \tag{3.36}$$

with

$$h_4 = -x_4'\ ,\quad h_1 = x_1 - x_1'\quad ,\quad h^2 = h_4^2 + h_1^2 + x_2^2;$$

For ρ_{24} one obtains a similar expression, which vanishes for $x_2 = 0$ and in the middle of the string, which exactly corresponds to the data of [25].

Finally, we can make a detailed comparison of the prediction for ρ_{14} in (3.36) with present lattice data. One obtains a simple analytic result for $\rho_{14}(x_2 \equiv x_\perp)$ in case of a very long string. The transverse shape measured at the middle is given by

$$\rho_{14} = \frac{2\pi a^2}{\delta^2}\left[D(0)(1+\delta x_2) - D_1(0)\frac{1}{2}(\delta x_2)^2\right]e^{-\delta x_2} \tag{3.37}$$

ρ_{14} was calculated as a function of x_1, x_2 from (3.36) keeping $D_1(0) = \frac{1}{3}D(0)$. One finds $\delta^{-1} \approx 0.2\,\mathrm{Fm}$, which is in good agreement with [24].
The results allow to predict all curves for other values of $x_\parallel$ and $x_\perp$: the agreement with the numerical results is very satisfactory [25, 26].

The asymptotics of the string profile at large $x_\perp$ is shown to be exponential, see eq. (3.37), just as the asymptotics of D, D_1, measured in [21, 22]. This in contrast to the behaviour inside the string, where the Gaussian–like flattening is observed. This effect is connected to the smearing effect due to integration in (3.36), yielding polynomial factors in (3.37). The radius of the string is close to $\delta^{-1} = T_g$ and hence gluon correlation length T_g has another physical meaning – the thickness of the confining string.

One can define higher irreducible correlator $\bar{\gamma}^c$ (the superscript ^{c} is written for *connected*, so the trace appears in (3.38) only once) as follows

$$\bar{\gamma}^c \approx a^4[\langle FF' \rangle_{q\bar{q}} - \langle FF' \rangle_0] \tag{3.38}$$

From (3.38) it is clear, that $\bar{\gamma}^c$ contains only double plaquette correlations. Most of the lattice data for $\bar{\gamma}^c$ are compatible with zero net effect, within two standard deviations (see [26] and recent analysis in [30]).

The same line of reasoning can be elaborated in the case of qqq states, i.e. baryons. The reader is referred to the paper [116], where the corresponding field profiles of the string were obtained. An interesting difference from the meson case considered so far is the vanishing of the average value of the CE field at the point of the string junction in the baryon, occuring due to symmetry reasons. This fact does not mean of course the vanishing of higher order correlators, measuring the fluctuations of the fields around the mean value (see [116]).

For the double correlator $\bar{\gamma}^c_{\mu\nu}$ one can use the non-abelian Stokes theorem and the cluster expansion, to connect $\bar{\gamma}^c$ to the quartic correlator G_4 and hence to estimate it from the lattice data. Representing the coordinate dependence of G_4 as an exponent similarly to $G_2 \equiv D(h^2)$ with the same δ, one gets the dimensionless ratio as

$$\frac{G_4(0)}{(G_2(0))^2} = \frac{\bar{\gamma}^c(0)}{(\rho(0))^2} \approx 2 - 3$$

From the point of view of the cluster expansion convergence, the expansion parameter is of the order $G_2(0)T_g^4 \ll 1$, and one finds that $G_4(0)T_g^8 \sim (2-3) \cdot (G_2(0)T_g^4)^2$ in agreement with the idea of the bilocal correlator dominance.

We turn now to the case of the disconnected correlators. The advantage of this probe is that it measures the field configurations induced by a static quark–antiquark pair which are gauge invariant by itself, i.e. in the simplest case the traces of the squares of the CE and CM field components: $\sum_a (E_i^a)^2$, $\sum_a (B_i^a)^2$. These expressions are directly related to physical quantities as for instance the energy density. We are interested in the difference between the squared field strength of the situation with a static quark-antiquark pair and the vacuum expectation value without such pair. This difference which we denote by $\Delta F^2_{\alpha\beta}(x)$ is given by:

$$\Delta F^2_{\alpha\beta}(x) \equiv \frac{4}{a^4} \frac{\langle \mathrm{Tr}\, W[\mathcal{C}]\ \mathrm{Tr}\, P_{\alpha\beta}(x) \rangle - \langle \mathrm{Tr}\, W[\mathcal{C}] \rangle \langle \mathrm{Tr}\, P_{\alpha\beta}(x) \rangle}{\langle \mathrm{Tr}\, W[\mathcal{C}] \rangle}. \tag{3.39}$$

Here Tr $P_{\alpha\beta}$ is the W–loop of a small square in the $\alpha - \beta$ plane. From it we obtain the desired gauge invariant squared field strengths:

$$\text{Tr } P_{\alpha\beta}(x) \;=\; N_c - \frac{1}{4}a^4 \sum_a F^a_{\alpha\beta}(x) F^a_{\alpha\beta}(x) \;+\; \mathcal{O}(a^8) \tag{3.40}$$

We see that we have to deal here with a more complex problem than in the previous case, namely the evaluation of the expectation value of two loops. Therefore the factorization hypothesis of the Gaussian model is tested here much more severely: vacuum expectation values contain at least four gluon field tensors which have to be reduced to two-point correlators by the factorization assumption discussed above.

We again apply the non-abelian Stokes theorem to the line integrals in the loops and transform then in surface integrals as discussed before. Expanding the exponentials in eq.(3.39) we obtain:

$$\begin{aligned}
\Delta F^2_{\alpha\beta}(x) \;=\; & \frac{4}{a^4}\frac{1}{\langle W\rangle}\Bigg(\sum_{n=1}^{\infty}(-i)^n \overbrace{\int\cdots\int}^{\text{surfaceordered}} d\sigma^W_{\mu_1\nu_1}\cdots d\sigma^W_{\mu_n\nu_n}\,\text{Tr}\,\left[t^{a_1}\cdots t^{a_n}\right]\times \\
& \int\!\!\int d\sigma^P_{\mu\nu} d\sigma^P_{\rho\sigma} \times \frac{(-i)^2}{2!}\text{Tr}\,\left[t^a t^b\right]\langle F^{a_1}_{\mu_1\nu_1}\cdots F^{a_n}_{\mu_n\nu_n}\, F^a_{\mu\nu}F^b_{\rho\sigma}\rangle \;- \\
& \sum_{n=1}^{\infty}(-i)^n \overbrace{\int\cdots\int}^{\text{surface}-\text{ordered}} d\sigma^W_{\mu_1\nu_1}\cdots d\sigma^W_{\mu_n\nu_n}\,\text{Tr}\,\left[t^{a_1}\cdots t^{a_n}\right]\int\!\!\int d\sigma^P_{\mu\nu}d\sigma^P_{\rho\sigma}\times \\
& \frac{(-i)^2}{2!}\text{Tr}\,\left[t^a t^b\right]\langle F^{a_1}_{\mu_1\nu_1}\cdots F^{a_n}_{\mu_n\nu_n}\rangle\,\langle F^a_{\mu\nu}F^b_{\rho\sigma}\rangle\Bigg)
\end{aligned} \tag{3.41}$$

The surfaces S_W and S_P with the elements $d\sigma^W$ and $d\sigma^P$ are bordered by the loop of the static quarks and the plaquette respectively and contain both the reference point y, the choice of these surfaces will be discussed later. The indices of the surface elements are restricted to $\mu < \nu$.

Using the factorization prescription given in the previous section in and subtracting out the terms where the fields on S_W and S_P factorize separately we obtain:

$$\begin{aligned}
\Delta F^2_{\alpha\beta}(x) = & \frac{4}{a^4}\frac{1}{\langle W\rangle}\Bigg(\sum_{n=1}^{\infty}(-i)^{2n} \overbrace{\int\cdots\int}^{\text{surfaceordered}} d\sigma^W_{\mu_1\nu_1}\cdots d\sigma^W_{\mu_{2n}\nu_{2n}}\times \\
& \text{Tr}\,\left[t^{a_1}\cdots t^{a_{2n}}\right]\int\!\!\int d\sigma^P_{\mu\nu}d\sigma^P_{\rho\sigma}\frac{(-i)^2}{2!2}\Bigg[\sum_{\text{pairs}}^{\alpha_1,\beta_1\cdots\alpha_n,\beta_n}\langle F^{a_{\alpha_1}}_{\mu_{\alpha_1}\nu_{\alpha_1}} F^{a_{\beta_1}}_{\mu_{\beta_1}\nu_{\beta_1}}\rangle\cdots
\end{aligned} \tag{3.42}$$

$$\langle F^{a_{\alpha_{n-1}}}_{\mu_{\alpha_{n-1}}\nu_{\alpha_{n-1}}} F^{a_{\beta_{n-1}}}_{\mu_{\beta_{n-1}}\nu_{\beta_{n-1}}}\rangle \times \langle F^{a_{\alpha_n}}_{\mu_{\alpha_n}\nu_{\alpha_n}} F^{a}_{\mu\nu}\rangle \, \langle F^{a_{\beta_n}}_{\mu_{\beta_n}\nu_{\beta_n}} F^{a}_{\rho\sigma}\rangle \Big]\Big).$$

Of the $2n$ field strengths running over the surface S_W two are correlated with a $F^C_{\mu\nu}$ on the pyramid of the plaquette and the other $(2n-2)$ among one another. We now use the prescription developed in the previous section and neglect (in any order of the expansion of the exponentials) all terms which are not fully ordered, i.e. which are suppressed by at least a factor T_g/L. For all fully ordered expressions the traces in (3.42) are identical and one obtains finally after a lengthy calculation [45]:

$$\Delta F^2_{\alpha\beta}\,(x) \;=\; \frac{1}{a^4}\frac{1}{2N_c(N_c^2-1)}\left[\int\int d\sigma^W_{\mu\nu} d\sigma^P_{\rho\sigma} \langle F^a_{\mu\nu} F^a_{\rho\sigma}\rangle\right]^2 \tag{3.43}$$

We note that in the integral one surface element is connected to the surface S_W (indicated by the index W) and one to the plaquette oriented in α,β-direction (indicated by P) As noted above the choice of the surfaces is somewhat arbitrary and depends on the way by which we transport colour flux from the point x to the reference point y. Choosing a straight line it is reasonable to put the reference point between the plaquette and the W-loop. In this case the surfaces are the sloping sides of pyramides with the reference point as apex. It was checked in [117], [45], that different choices of the surfaces leads only to moderate changes in the results. A reasonable strategy seems to be to choose the reference point in such a way that the total resulting surface is minimal.

For the evaluation of (3.43) we refer to the paper [117], [45] and quote only the main results in below. First of all, it can be seen by symmetry arguments that for that case the difference of the square of the magnetic field strengths, i.e. the plaquette $P_{\alpha\beta}(x)$ with no time (x_4) component, vanishes identically. This means that the CM background field is not affected by the static color charges.

The electric field perpendicular to the W–loop is also practically not affected but only the difference of the squared electric field parallel to the loop is. In both cases the squared electric field difference is negative, i.e. the presence of the static source diminishes the vacuum fluctuations

In Figs.6,7 we display the value of the squared field strength parallel to the spatial loop extension $\Delta F^2_{34}\,(x) = -\langle E^2_z(x_3,r)\rangle_{\mathrm{q\bar{q}-vacuum}}$ as a function of the coordinates $r=\sqrt{x_1^2+x_2^2}$ and x_3 (the results are rotational invariant around

the x_3-axes) for different spatial separations R_W (notice, that $a = 0.35$ fm on these pictures is a chosen scale of distance analogous to the correlation length T_g and has nothing to do with the lattice spacing a used above).

The squared field strength reaches its saturation value $\Delta F_{34}^2\,(x)$ which approximately equals to 14MeV/Fm^3 for a spatial extension of the W-loop $R_W \approx 4T_g$. The transversal extension, defined by

$$\bar{r} \equiv \sqrt{\frac{\int dr\, r\, r^2\, \Delta F_{34}^2\,(0,r)}{\int dr\, r\, \Delta F_{34}^2\,(0,r)}}, \tag{3.44}$$

is practically independent of R_W and about 1.8 times the correlation length T_g. As mentioned above, the quantity $\Delta F_{34}^2\,(x)$ is the difference between the squared field strength in the presence of a static $\bar{q}q$-pair and the non-perturbative vacuum field strength. One can construct several quantities of interest from it. For example, the figure 10 from the reference [117] displays the unsubtracted squared field tensor of the color source in units of the vacuum gluon condensate $\langle F^2 \rangle$, while the figures 11–13 from the same reference shows the squared field strength including the perturbative, i.e. Coulomb contribution.

The result is therefore very satisfying for a physical intuition: a CE flux tube is formed between the quark-antiquark pair whose energy content increases linearly with the separation of the pair. One may even compare the energy content of the flux tube formed in that way with the quark antiquark potential calculated with the help of a single W-loop. In order to do that we discuss two low energy theorems in the next subsection.

The analysis of higher cumulants contributions for the disconnected correlators was performed in [118]. It was shown, how quartic cumulant can modify the profile of the string and, in particular, lead to (small) nonzero components of the magnetic fields averages.

The pictures displayed on the Fig.5 and Figs.6,7 are rather similar, despite they were obtained by use of different probes, and both are in good agreement with the lattice results for the string profile. We found it instructive to demonstrate how FC formalism works in these two different settings: in the first case the mean value of the CE field in the presence of the static pair was calculated in the Gaussian approximation and its squared average was shown on the Fig.5; in the second case the correlation of two gauge-invariant object was taken and reduced in the Gaussian approximation to the integral of the square of the two-point correlator again. From general point of view

the typical correlation length, characterizing the decrease of the fields in the case of connected correlators is related to the mass of the gluelumps (and to T_g^{-1} in the Gaussian approximation), while that of the second case represent somehow the glueball mass, since it corresponds to the interaction of two white objects.

Summarizing, all the results discussed so far, together with the arguments on Casimir scaling supports the fundamentality of the bilocal correlator, which defines the nonperturbative dynamics of confinement. On the contrary, there are no data known to the authors up to now, which would indicate a significant role played by the highest field strength correlators.

3.5 Low energy theorems and consistency of the model

In this subsection we consider two low energy theorems and apply them to the results of the previous sections in order to check the consistency of the model. We follow here closely the line of reasoning given in [117]. Let us consider the vacuum expectation value of a W–loop:

$$\langle W[C]\rangle = \frac{1}{N}\langle \mathrm{Tr}\,\mathrm{P}\exp[i\int_C A_\mu dx_\mu]\rangle \tag{3.45}$$

Here C denotes a closed loop. Differentiating $\log\langle W[C]\rangle$ with respect to $-\frac{1}{2g^2}$ one obtains:

$$8\pi\alpha_{s0}^2 \frac{\partial \log[\,\langle W[C]\rangle\,]}{\partial \alpha_{s0}} = \frac{1}{\langle W[C]\rangle}\left\langle \int \mathrm{d}^4x \mathrm{Tr}\;[F_{\mu\nu}(x)F_{\mu\nu}(x)]\cdot\right.$$
$$\left.\mathrm{Tr}\;\mathrm{P}\exp[i\int_C A_\mu dx_\mu]\right\rangle - \left\langle \int \mathrm{d}^4x \mathrm{Tr}\;[F_{\mu\nu}(x)F_{\mu\nu}(x)]\right\rangle \tag{3.46}$$

If the loop C is a rectangle with spatial extension $-R/2 < x_3 < R/2$ and temporal extension $-T/2 < x_4 < T/2$ the right-hand side of (3.46) can be simplified further in the limit of large T. Let the length T_g be of the order of the correlation length of the gluon field strengths. Then for $|x_4| > T/2 + T_g$ the r.h.s. of (3.46) is zero and for $|x_4| < T/2 - T_g$ independent of x_4. This gives in the limit of large T:

$$8\pi\alpha_{s0}^2 \frac{\partial \log[\,\langle W[C]\rangle\,]}{\partial \alpha_{s0}} = \left\langle \int \mathrm{d}^3x \mathrm{Tr}\,[F_{\mu\nu}(x)F_{\mu\nu}(x)]\right\rangle_R \cdot T \tag{3.47}$$

Here and in the following x_4 is zero and the expectation value $\langle .. \rangle_R$ shall denote the expectation value in the presence of a static quark-antiquark pair at distance R, with the static sources in the fundamental representation, and the expectation value in absence of the sources has been subtracted.

The potential $V(R)$ is defined by (3.15) and thus one obtains the low energy theorem:

$$-4\pi\alpha_{s0}^2 \frac{\partial V(R)}{\partial \alpha_{s0}} = \frac{1}{2} \left\langle \int \mathrm{d}^3x \mathrm{Tr}\left[F_{\mu\nu}(x) F_{\mu\nu}(x)\right] \right\rangle_R \tag{3.48}$$

In (3.48) self energies are understood to be subtracted.

Up to now we have used the bare coupling g_0 and bare fields. If we use the background gauge fixing [121, 122, 123] we have

$$g = Z_g^{-1} g_0 \ , \ A_\mu = Z^{-1/2} A_\mu^{(0)} \ , \ Z_g Z^{1/2} = 1 \ , \ A_\mu = g_0 A_\mu^{(0)} = g A_\mu \tag{3.49}$$

where g and A_μ are renormalized quantities. The renormalized squared field strength tensor is given by:

$$\mathrm{Tr}\left[F_{\mu\nu}(x) F_{\mu\nu}(x)\right] = Z_{F^2}^{-1} \mathrm{Tr}\left[F_{\mu\nu}^{(0)}(x) F_{\mu\nu}^{(0)}(x)\right] \tag{3.50}$$

where Z_{F^2} was calculated [124] in background gauge with Landau gauge fixing to be $Z_{F^2} = 1 + \frac{1}{2} g_0 \frac{\partial}{\partial g_0} \log Z$ and thus we obtain

$$\frac{\partial \alpha_s}{\partial \alpha_{s0}} = Z Z_{F^2} \ . \tag{3.51}$$

We use (3.49)-(3.51) to express (3.48) in terms of renormalized quantities:

$$-\alpha_s \frac{\partial V(R)}{\partial \alpha_s} = \frac{1}{2} \left\langle \int \mathrm{d}^3x \mathrm{Tr}\left[F_{\mu\nu}(x) F_{\mu\nu}(x)\right] \right\rangle_R \tag{3.52}$$

In order to evaluate the left-hand side of (3.52) we use standard renormalization group arguments: the only scales are the renormalization scale μ and R, thus the potential $V = V(R, \mu, \alpha_S)$ satisfies on dimensional grounds:

$$\left(R \frac{\partial}{\partial R} - \mu \frac{\partial}{\partial \mu} \right) V(R, \mu, \alpha_s) = -V(R, \mu, \alpha_s) \tag{3.53}$$

But for V as a physical quantity the total derivative with respect to μ must vanish:

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}V(R,\mu,\alpha_s) = \left(\mu\frac{\partial}{\partial\mu} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right)V(R,\mu,\alpha_s) = 0 \tag{3.54}$$

where $\beta(\alpha_s) \equiv \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\alpha_s(\mu)$. We get from (3.53) and (3.54):

$$\frac{\partial V(R)}{\partial\alpha_s} = -\frac{1}{\beta}\left\{V(R) + R\frac{\partial V(R)}{\partial R}\right\} \tag{3.55}$$

and finally one obtains with (3.52) the low energy theorem

$$\left\{V(R) + R\frac{\partial V(R)}{\partial R}\right\} = \frac{1}{2}\frac{\beta}{\alpha_s}\langle\int \mathrm{d}^3x \mathrm{Tr}\,[F_{\mu\nu}(x)F_{\mu\nu}(x)]\rangle_R \tag{3.56}$$

or equivalently

$$\left\{V(R) + R\frac{\partial V(R)}{\partial R}\right\} = \frac{1}{2}\frac{\beta}{\alpha_s}\langle\int \mathrm{d}^3x \left(\vec{E}(x)^2 + \vec{B}(x)^2\right)\rangle_R \tag{3.57}$$

The (3.57) is for the case of a linear potential just one of the many low energy theorems derived in [85]. But there renormalization was only discussed to leading order.

The relation between the potential $V(R)$ and the energy density $T_{00}(x)$ is

$$V(R) = \int \mathrm{d}^3x\langle T_{00}(x)\rangle_R = \frac{1}{2}\langle\int \mathrm{d}^3x \left(-\vec{E}(x)^2 + \vec{B}(x)^2\right)\rangle_R \tag{3.58}$$

Note that we are in a Euclidean field theory, hence the minus sign of the electric field. Moreover on the r.h.s. of (3.57) and (3.58) we have renormalized composite operators.

In the model under consideration where the CM and the transversal component of the electric field vanishes the potential must be proportional to the spatial integral over $E_\parallel^2$. In view of the approximations made concerning path ordering as well as the arbitrarines of the choice of the reference point this is a very valuable consistency test (see details in [117]).

In lattice QCD (3.57) and (3.58) are known as Michael's sum-rules [125]. But in the derivation of the action sum-rule (3.57) scaling of $V(R)$ with R was not taken properly into account and hence the second term on the l.h.s. of (3.57) is missing in [125].

First consider the region where the potential is linear. Then (3.57) reads:

$$\frac{1}{2}\frac{\beta}{\alpha_s}\langle\int \mathrm{d}^3x \left(\vec{E}(x)^2+\vec{B}(x)^2\right)\rangle_R = 2\,V(R) \tag{3.59}$$

Since in the model of the stochastic vacuum the static sources do not modify the CM field, i.e. $\langle\vec{B}^2\rangle_R = 0$, we have consistency between (3.58) and (3.59) only if

$$\beta/\alpha_s = -2 \tag{3.60}$$

From the energy sum-rule (3.58) we have already obtained $\alpha_s = 0.57$, thus consistency of the MSV requires that (3.60) should be satisfied for this value of α_s.

The β-function can be expanded in a power series in α_s:

$$\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3 - \frac{\beta_2}{64\pi^3}\alpha_s^4 + ... \tag{3.61}$$

The n-loop expansion gives the series up to $\alpha_s^{\mathrm{n}+1}$. The first two terms of the power series are scheme independent, the numerical value of the term proportional to α_s^4 has been evaluated [126] in $\overline{MS}$. For pure gauge SU(3) theory we have [7]:

$$\beta_0 = 11, \quad \beta_1 = 51, \quad \beta_{2\overline{MS}} = 2857 \tag{3.62}$$

We see that the condition (3.60) yields $\alpha_s = 1.14$ using β on the one loop level, $\alpha_s = 0.74$ on the two loop level and $\alpha_s = 0.64$ on the three loop level in $\overline{MS}$. This value is already very close to the value of $\alpha_s = 0.57$ determined from the flux tube calculation using (3.58). If we use for β the series (3.61) truncated at order α_s^4 and impose condition (3.60) for $\alpha_s = 0.57$ and determine the coefficient β_2 accordingly we find $\beta_2 = 6250$ i.e. about twice the $\overline{MS}$ value.

It is also easy to see from the above that for distances $R/T_g \leq 1$, where $T_g \approx 0.3$ fm the MSV does no longer yield a linearly rising potential and hence we have

$$\left[V(R) + R\frac{\partial V(R)}{\partial R}\right] \neq 2V(R)\,, \qquad R/T_g \leq 1$$

Therefore the conditions (3.58) and (3.59) cannot be fulfilled simultaneously for $\langle\vec{B}^2\rangle_R = 0$. This does by no means speak against the model, since in the

derivation of the results it was stressed that the spatial extension of the loop had to be large as compared to the correlation length of the field strengths in order to justify the evaluation of $\vec{E}^2$ and $\vec{B}^2$.

Notice, that no use of the equations of motion has been made explicitly. The factorization hypothesis (Gaussian measure) enters crucially in the evaluation of the squared color fields. In view of the drastic approximations made it is gratifying that relation (3.60), which is a highly non-trivial consequence of the dynamics of the system, is so well fulfilled.

The scale μ at which the MSV works was found to be the one where $\alpha_s(\mu) = 0.57$ by using the energy sum-rule. With the help of the action sum-rule we now checked the consistency of the MSV at this scale and obtained an effective β-function.

As can be seen from (3.57) and (3.58) the squared field strengths $\vec{B}^2$ and $\vec{E}^2$ depend on the renormalization scale μ and hence on $\alpha_s(\mu)$. We may use relations (3.58) and (3.59) in order to predict the ratio of

$$Q \equiv \frac{\int \mathrm{d}^3x \langle \vec{B}(x)^2 \rangle_R}{\int \mathrm{d}^3x \langle \vec{E}(x)^2 \rangle_R} = \frac{2 + \beta/\alpha_s}{2 - \beta/\alpha_s}$$

as a function of the strong coupling α_s. This result can be checked in lattice gauge calculations.

4 Heavy quark dynamics and field correlators

In this chapter we apply the MFC to heavy quarkonia having in mind first of all bottomonium and charmonium and, qualitatively, $s\bar{s}$ states. For these systems one should take into account both perturbative and nonperturbative interactions, since the size of ground states is around 0.2 and 0.4 Fm for bottomonium and charmonium respectively, and the color Coulomb interaction is dominant below 0.3 Fm. As we shall see in this chapter the MFC in general and the Gaussian stochastic approximation in particular are very well adapted to the description of heavy quarkonia.

Actually MFC is here the only analytic method which takes into account both perturbative and nonperturbative interaction at all distances. The first account for the effects of finite correlation length T_g was made in [22] in attempt to correct the unphysical n^6-growth of NP energy shift obtained in [60, 61]. For states of large temporal extent (here belong all states of $c\bar{c}$ and $s\bar{s}$) the potential picture emerges from MFC with potentials expressed through the only two correlators D and D_1. For states of small size the NP configurations can be accounted for in form of condensates, (see Voloshin [60] and Leutwyler [61]). The general situation was discussed in [35, 39].

Various comparisons and cross-checks can be made with the MFC results. First, one compares with numerous experimental data and finds in most cases good agreement; second, lattice MC data for scalar and spin-dependent potentials are available; third, phenomenological potential models in conjunction with the $O(\alpha_s^2)$ perturbative results are checked versus MFC predictions. The reader is also referred to the review [127] where different aspects of the heavy quark dynamics in the confining vacuum are extensively discussed.

The material of present chapter is based mostly on papers [35, 38] and lectures [59] and the paper [128].

The structure of the chapter is as follows. In section 4.1 the general formalism of the Feynman–Schwinger representation (FSR) for the quark–antiquark Green's function is presented and the nonrelativistic approximation is deduced. In section 4.2 the NP and perturbative parts of static potential are derived from the MVC. In section 4.3 the spin-dependent potentials are obtained from MVC and compared to the lattice data. Finally, in section 4.4 results of numerical calculations and general discussion are given.

4.1 The general formalism for the quark–antiquark Green's function

We start with the quark Green's function in the form of the proper–time and path integral (the Feynman–Schwinger representation [35, 36, 37, 38, 39, 128])

$$S(x,y) = i(m-\hat{D})\int_0^\infty ds \int Dz\, e^{-K}\Upsilon(x,y) \tag{4.1}$$

where Υ contains spin insertions into parallel transporter

$$\Upsilon(x,y) = \mathrm{P}_A\,\mathrm{P}_F\,\exp[i\int_y^x A_\mu dz_\mu + \int_0^s d\tau\, g\Sigma F(z(\tau))], \tag{4.2}$$

$$K = m^2 s + \frac{1}{4}\int_0^s \dot{z}_\mu^2 d\tau,$$

and double ordering in A_μ and $F_{\mu\nu}$ is implied by operators P_A, P_F.

We have also introduced the 4×4 matrix in Dirac indices

$$\Sigma F \equiv \vec{\sigma}_i \begin{pmatrix} \vec{B}_i & \vec{E}_i \\ \vec{E}_i & \vec{B}_i \end{pmatrix} \tag{4.3}$$

Neglecting spins one has instead of (4.2)

$$\Upsilon(x,y) \to \Phi(x,y) \equiv \mathrm{P}\exp\; i\int_y^x A_\mu dz_\mu \tag{4.4}$$

In terms of $(q\bar{q})$ Green's functions (4.1) and initial and final state matrices Γ_i, Γ_f (such that $\bar{q}(x)\Gamma_f\Phi(x,\bar{x})q(\bar{x})$ is the final $q\bar{q}$ state) the total relativistic gauge–invariant $q\bar{q}$ Green's function in the quenched approximation is

$$G(x\bar{x}, y\bar{y}) = \langle \mathrm{Tr}\,(\Gamma_f S_1(x,y)\Gamma_i \Phi(y,\bar{y}) S_2(\bar{y},\bar{x})\Phi(\bar{x},x))\rangle$$
$$-\,\langle \mathrm{Tr}\,(\Gamma_f S_1(x,\bar{x})\Phi(\bar{x},x))\mathrm{Tr}\,(\Gamma_i S_2(\bar{y},y)\Phi(y,\bar{y}))\rangle \tag{4.5}$$

The angular brackets in (4.5) imply averaging over gluon field A_μ.

The second term on the r.h.s. of (4.5) represents a mixing of a flavour singlet quarkonium with two– or three–gluon glueball. Except for an accidental degeneration of masses, this term is a small correction and will be omitted in what follows.

Since we are interested primarily in heavy quarkonia, it is reasonable to do a systematic nonrelativistic approximation. To this end we introduce the

real evolution parameter (time) t instead of the proper time τ in K, ($\bar\tau$ in $\bar K$) and dynamical mass parameters $\mu, \bar\mu$

$$\frac{dt}{d\tau} = 2\mu_1, \ \frac{dt}{d\bar\tau} = 2\mu_2; \quad \int_0^s \dot z_\mu^2(\tau) d\tau = 2\mu_1 \int_0^T dt \left(\frac{dz_\mu(t)}{dt}\right)^2 \tag{4.6}$$

Here we have denoted $T \equiv (x_4 + \bar x_4)/2$. Nonrelativistic approximation means, that one writes for $z_4(t), \bar z_4(t)$

$$z_4(t) = t + \zeta(t), \quad 2\mu_1 = \frac{T}{s_1}; \tag{4.7}$$

$$\bar z_4(t) = t + \bar\zeta(t), \quad 2\mu_2 = \frac{T}{s_2}$$

and expands in fluctuations $\zeta, \bar\zeta$, which are $\mathcal{O}(\frac{1}{\sqrt m})$. Note that the integration in $ds_1 ds_2$ goes over into $d\mu_1 d\mu_2$. Physically expansion (4.7) means that we neglect in lowest approximation trajectories with backtracking of $z_4, \bar z_4$, i.e. neglect $q\bar q$ pair creation. One can persuade oneself that insertion of (4.7) into $K, \bar K$ allows to determine μ_1, μ_2 from the extremum in $K, \bar K$ to be

$$\mu_1 = m_1 + \mathcal{O}(1/m_1), \quad \mu_2 = m_2 + \mathcal{O}(1/m_2) \tag{4.8}$$

and one can further make a systematic expansion in powers $1/m_i$. (For details see [35]). At least to lowest orders in $1/m_i$ this procedure is equivalent to the standard (gauge–noninvariant) nonrelativistic expansion.

Let us keep the leading term of this expansion

$$G(x\bar x, y\bar y) = 4m_1 m_2 e^{-(m_1+m_2)T} \int D^3 z D^3 \bar z e^{-K_1 - K_2} \langle W(C)\rangle \tag{4.9}$$

where $K_1 = \frac{m_1}{2}\int_0^T \dot z_i^2(t) dt, \quad K_2 = \frac{m_2}{2}\int_0^T \dot{\bar z}_i^2(t) dt$ and $\langle W(C)\rangle$ is the W–loop operator with closed contour C comprising q and $\bar q$ paths, and initial and final state parallel transporters $\Phi(x, \bar x)$ and $\Phi(y, \bar y)$.

The representation (4.9) is our main object of study in next section.

4.2 Perturbative and nonperturbative static potentials

The technical tool we are using is the background perturbation theory. We represent the total gluon field A_μ as

$$A_\mu = B_\mu + a_\mu \tag{4.10}$$

where B_μ is the NP background, while a_μ is perturbative fluctuation. The principle of separation in (4.10) is immaterial for our purposes (for more discussion of this point see [129]).

For static charges trajectories are fixed straight lines and $\langle W(C)\rangle$ in (4.9) factorizes out defining the potential between static charges $V(R)$

$$\langle W(C)\rangle = \exp(-V(R)T) \tag{4.11}$$

where R is the distance between charges and T is the time distance in the rectangular loop; $C = R\times T$.

In the approximation, when only bilocal correlators are kept, $V(R)$ is given as [38, 39]

$$V_0(R) = C_D\left(-\frac{\alpha_s(R)}{R} + 2R\int_0^R d\lambda\int_0^\infty d\nu D(\lambda,\nu) + \right.$$
$$\left. + \int_0^R \lambda d\lambda \int_0^\infty d\nu(-2D(\lambda,\nu) + D_1(\lambda,\nu))\right) \tag{4.12}$$

In addition one also obtains radiative corrections due to transverse gluon exchange, which have been computed earlier [130].

Thus one has the resulting potential

$$V(R) = V_0(R) + \Delta V(R) \tag{4.13}$$

where $V_0(R)$ is defined in (4.12), and $\Delta V(R)$ is the sum of higher–order terms, namely

$$\Delta V(R) = \Delta V_1(R) + \Delta V_2(R) + \Delta V_3(R),$$

where $\Delta V_1(R)$ refers to $\mathcal{O}(\alpha_s^2)$ perturbative contributions [131, 132], $\Delta V_2(R)$ is interference term [133] which is $\mathcal{O}(\alpha_s)$ and $\Delta V_3(R)$ refers to contribution of higher FC.

4.3 Spin–dependent potentials

As was shown in the previous section, the leading term in the heavy mass expansion is the nonrelativistic form containing the W–loop with the contour C, formed by straight–line paths without spin insertions (ΣF), Eq. (4.9). To keep spin dependent terms we shall use (4.2), (4.5) and derive from it the

nonrelativistic expression in the limit when $m_i \to \infty$ and also $|\vec{x} - \vec{y}| \ll |x_0 - y_0|$.

Now one can use the standard definitions of Eichten, Feinberg and Gromes (EFG) [134, 135] for the SD potentials

$$V_{SD}(R) = \left(\frac{\sigma_1 \mathbf{L}_1}{4m_1^2} - \frac{\sigma_2 \mathbf{L}_2}{4m_2^2}\right)\left(\frac{1}{R}\frac{d\varepsilon}{dR} + \frac{2dV_1(R)}{RdR}\right) +$$

$$+ \frac{\sigma_2 \vec{L}_1 - \sigma_1 \vec{L}_2}{2m_1 m_2}\frac{1}{R}\frac{dV_2(R)}{dR} + \frac{\sigma_1\sigma_2 V_4(R)}{12 m_1 m_2} + \frac{(3\sigma_1 \mathbf{R} \sigma_2 \mathbf{R} - \sigma_1\sigma_2 R^2)V_3(R)}{12 m_1 m_2 R^2} \quad (4.14)$$

and identify $V_i(R)$ by expanding the exponents of $(\Sigma_i F)$ in (4.2) to the first order and keeping m^{-2} terms as compared to the leading spin–independent (static) contribution.

In Gaussian (bilocal) approximation one obtains (we denote quantities taken in Gaussian approximation by primes)

$$\frac{1}{R}\frac{dV_1'}{dR} = -\int_{-\infty}^{\infty} d\nu \int_0^R \frac{d\lambda}{R}\left(1 - \frac{\lambda}{R}\right) D(\lambda, \nu), \quad (4.15)$$

$$\frac{1}{R}\frac{dV_2'}{dR} = \int_{-\infty}^{\infty} d\nu \int_0^R \frac{\lambda d\lambda}{R^2}\left[D(\lambda,\nu) + D_1(\lambda,\nu) + \lambda^2 \frac{\partial D_1}{\partial \lambda^2}\right], \quad (4.16)$$

$$\frac{1}{R}\frac{d\varepsilon'}{dR} = \frac{1}{R}\int_{-\infty}^{\infty} d\nu \left(\frac{R}{2} D_1(R,\nu) + \int_0^R d\lambda D(\lambda,\nu)\right), \quad (4.17)$$

$$V_3 = -\int_{-\infty}^{\infty} d\nu R^2 \frac{\partial D_1(R,\nu)}{\partial R^2}, \quad (4.18)$$

$$V_4 = \int_{-\infty}^{\infty} d\nu \left(3D(R,\nu) + 3D_1(R,\nu) + 2R^2 \frac{\partial D_1}{\partial R^2}\right) \quad (4.19)$$

It is very simple to check, as it was done in [37], that the Gromes relation

$$V_1' - V_2' + \varepsilon' = 0 \quad (4.20)$$

is identically satisfied in bilocal approximations for ε', V_1', V_2' given by (4.15), (4.16) and (4.17) for the arbitrary choice of D and D_1 falling off at large distances.

One can also check that (4.17) is the derivative of the NP part of the scalar potential (4.12) (without Coulomb part).

It is interesting that (4.15) and (4.16) yield correct results in lowest order of perturbation theory, namely V_1' has no contribution and $\frac{V_2'}{R}$ is equal to $\frac{4\alpha_s}{3R^3}$, when the perturbative value of D_1, $D_1^{pert}(x) = 16\alpha_s/(3\pi x^4)$, is used.

4.4 Charmonium and bottomonium spectra.

In previous sections the scalar and spin–dependent potentials have been expressed through the Gaussian FC, functions D and D_1. The use of these potentials in the Schrödinger equation allows to calculate the spectra of charmonium and bottomonium and compare results with experimental data. The contents of this section is mostly based on [136].

The static potential according to can be written as

$$V_0(r) = V^{pert}(r) + \varepsilon(r) \tag{4.21}$$

The NP component of D, D_1 are taken from lattice measurements [24] in the form (see Section 2)

$$D(x) = D(0)e^{-|x|/T_g}, \quad D_1(x) = D_1(0)e^{-|x|/T_g} \tag{4.22}$$

where $D(0) \approx 3D_1(0)$.

Parameters used above for computations and corresponding to lattice results [22, 24] are

$$\begin{aligned} \sigma = 0.185\,\mathrm{GeV}^2, \quad \sigma_1 = \frac{1}{\pi}D_1(0)T_g^2 = 0.0064\,\mathrm{GeV}^2, \mathrm{GeV} \\ T_g^{-1} = 1\,\mathrm{GeV}, \quad m_b = 4.85\,\mathrm{GeV}, \quad m_c = 1.45\,\mathrm{GeV} \end{aligned} \tag{4.23}$$

The perturbative part of potential takes into account the asymptotic freedom and freezing at large distances with $\alpha_s = \alpha_s^{max} = 0.5$ [31, 32, 33].

Results of computations for the parameters of Eq. (4.23) are listed below in the Tables 1-2 together with experimental data and calculations with the Cornell potential [137].

Table 1

Masses of $b\bar{b}$ for FC (4.22) in comparison with experiment and those of Cornell potential [137]

State	Exper.	FC	Cornell
$\gamma(1S)$	9460	9460	" fit"
$\gamma(2S)$	10023	10023	10052
$\gamma(3S)$	10355	10347	10400
$\chi_b(1P)$	9900	9918	9960
$\chi_b(2P)$	10260	10250	10314
$\gamma(2S) - \gamma(1S)$	563	563	592
$\gamma(3S) - \gamma(2S)$	332	324	345
$\gamma(2S) - \chi_b(1P)$	123	105	93
$\chi_b(2P) - \chi_b(1P)$	359	332	354

Table 2

Masses of $c\bar{c}$ for FC (4.22) *vs* experiment

State	Exper.	FC	Cornell
$J\psi$	3097	3096	3097
$\Psi(2S)$	3686	3682	3688
$\Psi(3S)$	4040	4128	4111
$\chi_{cog}(1P)$	3525	3516	3525
$\Psi(1D)$	3770	3800	3810
$\Psi(2S) - J/\psi'$	589	586	591
$\Psi(2S) - \chi_{cog}(1P)$	161	166	163
$\Psi(1D) - J/\psi'$	673	705	713

Results listed in Tables 1-2 demonstrate several successes of MFC in comparison with the standard potential models like Cornell. Namely, **i)** it is possible to describe bottomonium spectrum with the current b-quark mass, $m_b = 4.8\,GeV$, while in Cornell potential an effective mass $m_b = 5.17\,GeV$ is used; **ii)** a good description of leptonic width is obtained (not shown in Tables 1,2, see [136]); **iii)** the overal agreement of levels with experiment is rather good and certainly better than for the Cornell potential.

The fine structure of the spectrum is much more delicate issue. In particular, here one should use the $\mathcal{O}(\alpha_s^2)$ contributions to LS and scalar interaction. The full analysis done in [138] demonstrates a good description of charmonium and also checks the consistency of the model with the lattice data on spin-dependent potentials [139].

5 Field correlators at $T > 0$ and deconfinement transition

The finite temperature QCD provides a unique insight into the structure of the QCD vacuum, in particular it gives an important information about the mechanism of confinement. It is also of more practical interest, since the nature of the deconfinement transition has its bearing on the cosmology and the hot QCD plasma can possibly be tested in heavy ion collisions.

From theoretical point of view the hot QCD is a unique theoretical laboratory where both perturbative (P) and nonperturbative (NP) methods can be applied in different temperature regimes.

It is believed that the perturbative QCD is applicable in the deconfined phase at large enough temperatures T, where the effective coupling constant $g(T)$ is small [140, 141], while at small T (in the confined phase) the NP effects instead are most important. However even at large T the physics is not that simple: some effects, like screening (electric gluon mass), need a resummation of the perturbative series [142, 143], while the effects connected with the magnetic gluon mass demonstrate the infrared divergence of the series [144]. For the role of hadron degrees of freedom in the transition see e.g. [145].

During last years there appeared a lot of lattice data which point at the NP character of dynamics above T_c. Here belong: **i)** area law of spatial W–loops [146, 147], **ii)** screening masses of mesons and baryons [148, 149] and glueballs [150], **iii)** temperature dependence of Polyakov–line correlators [151].

In addition, behaviour of $\epsilon - 3p$ above T_c has a bump incompatible with the simple quark–gluon gas picture [152].

Thus the inclusion of NP configurations into analysis at $T > 0$ and also at $T > T_c$ is necessary. This was done using background field method in [153]. Besides defining NP dynamics, the NP configurations also ensure freezing of α_s and stabilize perturbative series [153]. To describe the phase transition a simple choice of deconfined phase was suggested [154] where all NP CM configurations are kept intact as in the confined phase, whereas CE correlators responsible for confinement, vanish.

Recently this picture of phase transition has been tested and confirmed in the lattice calculations [25, 26] of electric and magnetic correlators. It was shown in [27, 28, 29] that the confining part of the electric correlator D^E

decreases steeply right above T_c, while the deconfining electric part, D_1^E, and magnetic part D^B, D_1^B, stay approximately constant. see Figs.2,3. Thus one has a consistent mechanism describing the structure of the QCD vacuum at $T \geq 0$ and the phase transition at $T = T_c$. This picture together with the background perturbation theory forms a basis of quantitative calculations, where field correlators (condensates) are used as the NP input [154].

The plan of the Section is as follows. In subsection 5.1 the modified background field formalism [153] is presented, based on the familiar background field method [121, 122, 123] for $T > 0$ and incorporating the 't Hooft's identity for integration over quantum fluctuations as well as over the background fields. The temperature phase transition is discussed in subsection 5.2 and resulting predictions for T_c are compared with lattice data. In subsection 5.3 the spacial W–loops are computed both below and above T_c.

5.1 Background field formalism

We derive here basic formulas for the partition function, free energy and Green's function in the NP background formalism at $T > 0$ [154]. The total gluon field A_μ is split into a perturbative part a_μ and NP background B_μ

$$A_\mu = B_\mu + a_\mu \tag{5.1}$$

where both B_μ and a_μ are subject to periodic boundary conditions. The principle of this separation is immaterial for our purposes here, and one can average over fields B_μ and a_μ independently

$$Z = \int DA_\mu \, \exp(-S(A)) = \frac{\int DB_\mu \eta(B) \int Da_\mu \exp(-S(B+a))}{\int DB_\mu \eta(B)} \tag{5.2}$$

$$\equiv \langle \langle \exp(-S(B+a)) \rangle_a \rangle_B$$

with the weight $\eta(B)$. In our case we choose $\eta(B)$ to fix field correlators and string tension at their observed values.

The partition function can be written as

$$Z(V, T, \mu = 0) = \langle Z(B) \rangle_B$$

where

$$Z(B) = N \int Da_\mu \, \exp(-\int_0^\beta d\tau \int d^3x \, L(x, \tau)) \tag{5.3}$$

where ϕ denotes all set of fields a_μ, Ψ, Ψ^+; N is a normalization constant, and the brackets $\langle ... \rangle_B$ means some averaging over (nonperturbative) background fields B_μ with the weight $\eta(B)$, as in (5.2). Explicitly

$$S(B+a) = \int_0^\beta d\tau \int d^3x \sum_{i=1}^{8} L_i,$$

where

$$\begin{aligned} L_1 = \frac{1}{4}(F^a_{\mu\nu}(B))^2;\ L_2 = \frac{1}{2} a^a_\mu W^{ab}_{\mu\nu} a^b_\nu, \\ L_3 = \bar{\Theta}^a (D^2(B))_{ab} \Theta^b;\ L_4 = -ig\bar{\Theta}^a (D_\mu, a_\mu)_{ab} \Theta^b \\ L_5 = \frac{1}{2}\alpha(D_\mu(B)a_\mu)^2;\ L_6 = L_{int}(a^3, a^4) \\ L_7 = -a_\nu D_\mu(B) F_{\mu\nu}(B);\ L_8 = \Psi^+(m + \hat{D}(B+a))\Psi \end{aligned} \tag{5.4}$$

Notice, that we have redefined the fields according to $A_\mu \to gA_\mu$ in this section.

In (5.5) $\bar{\Theta}, \Theta$ are ghost fields, α is gauge–fixing constant, L_6 contains 3– and 4–gluon vertices. For the fields B_μ satisfying classical equations of motion $L_7 = 0$.

The inverse gluon propagator in the background gauge is

$$W^{ab}_{\mu\nu} = -D^2(B)_{ab} \cdot \delta_{\mu\nu} - 2gF^c_{\mu\nu}(B) f^{acb} \tag{5.5}$$

where

$$(D_\lambda)_{ca} = \partial_\lambda \delta_{ca} - igT^b_{ca} B^b_\lambda \equiv \partial_\lambda \delta_{ca} - g f_{bca} B^b_\lambda \tag{5.6}$$

We consider first the case of pure gluodinamics, $L_8 \equiv 0$. Integration over ghost and gluon degrees of freedom in (5.3) yields

$$\begin{aligned} Z(B) = N'(\mathrm{Det} W(B))^{-1/2}_{reg} \left[\mathrm{Det}(-D_\mu(B) D_\mu(B+a))\right]_{a=\frac{\delta}{\delta J}} \times \\ \times \left\{ 1 + \sum_{l=1}^{\infty} \frac{S^l_{int}}{l!} \left(a = \frac{\delta}{\delta J} \right) \right\} \exp\left(-\frac{1}{2} J W^{-1} J \right)_{J_\mu = D_\mu(B) F_{\mu\nu}(B)} \end{aligned} \tag{5.7}$$

One can consider strong background fields, so that gB_μ is large (as compared to Λ^2_{QCD}), while $\alpha_s = \frac{g^2}{4\pi}$ in that strong background is small at all distances [31, 32]. In this case (5.7) is a perturbative sum in powers of g^n,

arising from expansion in $(ga_\mu)^n$. In what follows we shall discuss the Feynman graphs for the free energy $F(T)$, connected to $Z(B)$ via

$$F(T) = -T\ \log\langle Z(B)\rangle_B \tag{5.8}$$

As will be seen, the lowest order graphs already contain a nontrivial dynamical mechanism for the deconfinement transition. To the lowest order in ga_μ the partition function and free energy are

$$Z_0 = \langle \exp(-F_0(B)/T)\rangle_B$$

$$F_0(B)/T = \frac{1}{2}\log\ \mathrm{Det}W - \log\ \mathrm{Det}(-D^2(B)) =$$
$$= \mathrm{Tr}\int_0^\infty \zeta(t)\frac{dt}{t}\left(-\frac{1}{2}e^{-tW} + e^{tD^2(B)}\right) \tag{5.9}$$

where $\hat{W} = -D^2(B) - 2g\hat{F}$ and $D^2(B)$ is the inverse gluon and ghost propagator respectively, $\zeta(t)$ is a regularizing factor [154].

The ghost propagator can be written as [154, 155, 156]

$$(-D^2)^{-1}_{xy} = \langle x|\int_0^\infty ds\ \exp(sD^2(B))|y\rangle = \int_0^\infty ds(Dz)^w_{xy}e^{-K}\Phi(x,y) \tag{5.10}$$

where winding path integral is introduced [154]

$$(Dz)^w_{xy} = \lim_{N\to\infty}\prod_{m=1}^{N}\frac{d^4\zeta(m)}{(4\pi\varepsilon)^2}\sum_{n=0,\pm1,..}^{\infty}\int\frac{d^4p}{(2\pi)^4}\ e^{ip\left(\sum\zeta(m)-(x-y)-n\beta\delta_{\mu4}\right)} \tag{5.11}$$

with $\beta = 1/T$. For the gluon propagator an analogous expression holds true, expect that in (5.5) one should insert gluon spin factor $\mathrm{P}_F \exp\ 2g\hat{F}$ inside $\Phi(x,y)$. For a quark propagator the sum over windings acquires the factor $(-1)^n$ and quark spin factor is $\exp\left(g\sigma_{\mu\nu}F_{\mu\nu}\right)$ [154].

5.2 Temperature phase transition in QCD

We are now in position to make expansion of Z and F in powers of ga_μ (i.e. perturbative expansion in α_s), and the leading–nonperturbative term Z_0, F_0 – can be represented as a sum of contributions with different powers of N_c and we systematically will keep the leading terms $\mathcal{O}(N_c^2), \mathcal{O}(N_c)$ and $\mathcal{O}(N_c^0)$.

To describe the temperature phase transition one should specify phases and compute free energy. For the confining phase to lowest order in α_s

free energy is given by (5.3) plus contribution of energy density ε at zero temperature

$$F(1) = \varepsilon V_3 - \frac{\pi^2}{30} V_3 T^4 - T \sum_s \frac{V_3 (2m_\pi T)^{3/2}}{8\pi^{3/2}} \exp(-\frac{m_\pi}{T}) + \mathcal{O}(1/N_c) \quad (5.12)$$

where ε is defined by scale anomaly

$$\varepsilon \simeq -\frac{11}{3} N_c \frac{\alpha_s}{32\pi} \langle (F^a_{\mu\nu}(B))^2 \rangle \quad (5.13)$$

and the next terms in (5.12) correspond to the contribution of mesons (we keep only pion gas) and glueballs. Note that $\varepsilon = \mathcal{O}(N_c^2)$ while two other terms in (5.12) are $\mathcal{O}(N_c^0)$.

For the second phase (to be the high temperature phase) we had made an assumption (which was proved on the lattice [27, 28]) that all CM field correlators are the same as in the first phase, while all CE fields vanish. Since at $T = 0$ CM correlators and CE correlators are equal due to the Euclidean $O(4)$ invariance, one has

$$\langle (F^a_{\mu\nu}(B))^2 \rangle = \langle (F^a_{\mu\nu})^2 \rangle_{el} + \langle (F^a_{\mu\nu})^2 \rangle_{magn}; \ \langle F^2 \rangle_{magn} = \langle F^2 \rangle_{el} \quad (5.14)$$

The string tension σ which characterizes confinement is due to the electric fields [16, 17], e.g. in the plane $(i4)$ one has

$$\sigma = \sigma_E = \frac{g^2}{2} \int\int d^2x \langle \mathrm{Tr}\ E_i(x) \Phi(x,0) E_i(0) \Phi(0,x) \rangle + ... \quad (5.15)$$

where dots imply higher order terms in E_i (notice, that we have redefined the fields $A \to gA$ in this chapter with respect to the notations adopted in Section 3.)

Vanishing of σ_E liberates gluons and quarks, which will contribute to the free energy in the deconfined phase by their closed loop terms (5.11) with all possible windings while the magnetic correlators enter via perimeter contribution. As a result one has for the high-temperature phase (phase 2) (cf.[154]).

$$F(2) = \frac{1}{2} \varepsilon V_3 - (N_c^2 - 1) V_3 \frac{T^4 \pi^2}{45} \Omega_g - \frac{7\pi^2}{180} N_c V_3 T^4 n_f \Omega_q + \mathcal{O}(N_c^0) \quad (5.16)$$

where Ω_q and Ω_g are perimeter terms for quarks and gluons respectively, the latter was estimated in [157] from the adjoint Polyakov line; in what follows we replace Ω by one for simplicity.

Comparing (5.12) and (5.16), $F(1) = F(2)$ at $T = T_c$, one finds in the order $\mathcal{O}(N_c)$, disregarding all meson and glueball contributions

$$T_c^4 = \frac{\frac{11}{3} N_c \frac{\alpha_s \langle F^2 \rangle}{32\pi}}{\frac{2\pi^2}{45}(N_c^2 - 1) + \frac{7\pi^2}{90} N_c n_f} \tag{5.17}$$

For standard value of $G_2 \equiv \frac{\alpha_s}{\pi}\langle F^2 \rangle = 0.012\,\mathrm{GeV}^4$ (note that for $n_f = 0$ one should use larger value of G_2 [69]) one has for $SU(3)$ and different values of $n_f = 0, 2, 4$ respectively $T_c =$ 240, $150, 134\,\mathrm{MeV}$. This should be compared with lattice data [152] $T_c^{lat} =$ 240, $146, 131\,\mathrm{MeV}$. Agreement is quite good. Note that at large N_c one has $T_c = \mathcal{O}(N_c^0)$ i.e. the resulting value of T_c doesn't depend on N_c in this limit. Hadron contributions to T_c are $\mathcal{O}(N_c^{-2})$ and therefore suppressed if T_c is below the Hagedorn temperature as it typically happens in string theory estimates [158].

Till now we disregarded all perturbative and nonperturbative corrections to $F(2)$ except for magnetic condensate, the term $\frac{1}{2}\varepsilon V_3$. If we disregard also this term, considering in this way only free gas of gluons and quarks for the phase 2, we come to the model, considered in [159]. The values of T_c obtained in this way differ from ours not much – they are factor of $2^{1/4} \approx 1.19$ larger, but one immediately encounters problems with explanation of spatial string tension, screening masses etc., which are naturally are accounted for by the notion of magnetic confinement – nonzero values of magnetic correlators in the phase 2, including magnetic condensate term, $1/2\varepsilon V_3$.

Notice, that NP corrections may contribute to Ω_g, Ω_q. Their phenomenological necessity can be seen in the measured values of $\varepsilon - 3p$, see the corresponding pictures in [151, 152]. In case of $\Omega_g = \Omega_q = 1$ the difference $\varepsilon - 3p$ should be zero, and of course higher orders in N_c^{-1}(NP effects) and higher orders in g (perturbative effects) contribute to it. In [157] the authors tried to estimate effect of nonzero $(\Omega_g - 1)$, which is $\mathcal{O}(N_c^2)$, on the energy density and pressure. To this end the adjoint Polyakov line was used and the NP perimeter contribution was separated from it.

5.3 Spacial W–loops

In this section we derive area law for spatial W–loops, expressing spatial string tension in terms of CMC.

In standard way one obtains the area law for large W–loops of size L,

$L \gg T_g^{(m)}$ ($T_g^{(m)}$ is the magnetic correlation length)

$$\langle W(C)\rangle_{spacial} \approx exp(-\sigma_s S_{\min}) \tag{5.18}$$

where the spacial string tension is [154]

$$\sigma_s = \frac{g^2}{2}\int d^2x \langle\langle B_n(x)\Phi(x,0)B_n(0)\Phi(0,x)\rangle\rangle + \mathcal{O}(\langle B^4\rangle) \tag{5.19}$$

and n is the component normal to the plane of the contour, while the last term in (5.19) denotes contribution of the fourth and higher order cumulants. On general grounds one can write for the integrand in (5.19)(see also section 2):

$$\langle\langle B_i(x)\Phi(x,0)B_j(0)\Phi(0,x)\rangle\rangle = \delta_{ij}\left(D^B(x) + D_1^B(x) + \vec{x}^2\frac{\partial D_1^B}{\partial x^2}\right) - x_i x_j \frac{\partial D_1^B}{\partial x^2}, \tag{5.20}$$

and only the term $D^B(x)$ enters in (5.19)

$$\sigma_s = \frac{g^2}{2}\int d^2x D^B(x) + \mathcal{O}(\langle B^4\rangle) \tag{5.21}$$

similarly for the temporal W–loop in the plane ($i4$) one has the area law for $T < T_c$ with temporal string tension

$$\sigma_E = \frac{g^2}{2}\int d^2x D^E(x) + \mathcal{O}(\langle E^4\rangle) \tag{5.22}$$

For $T = 0$ due to the $O(4)$ invariance CE and CM correlators coincide and $\sigma_E = \sigma_s$. The lattice measurements of D_E and D_B [25, 26] show that σ_E and σ_S stay approximately constant in the whole interval $0 \le T \le T_c$, as it is suggested by the phase transition mechanism described above. For $T > T_c$ in the deconfining phase CEC vanish, while CMC stays nonzero and change on the scale of the dilaton mass $\sim$ 1 GeV, therefore one expects that σ_s stays intact till the onset of the dimensional reduction mechanism. This expectation is confirmed by the lattice simulation — σ_s stays constant up to $T \approx 1.4\,T_c$ [160]. Lattice data [160, 161] show an increase of σ_s at $T \approx 2\,T_c$, for $SU(2)$ which could imply the early onset of dimensional reduction.

It was shown how the Feynman–Schwinger representation (FSR) which proved to be very useful for $T = 0$, is modified for $T > 0$. In particular, all gluon and quark propagators are written as sums of path integrals with winding paths in the 4th direction. Using FSR the Hamiltonian for screening states of mesons and glueballs is derived, in [162] and screening masses and wave functions for lowest meson and glueball states are numerically computed. All this exemplifies NP background field theory and in particular MFC as a powerful instrument both for $T < T_c$ and for $T > T_c$.

6 Conclusions

The material of previous chapters contains only a small part of results obtained with the method of FC. The aim of the present review is to explain foundations of the method and possible lines of development.

In all fields where NP configurations are important the method of FC can be used to give a systematic description and calculation of effects. Here is the list of some results obtained heretofore with FC:

- Proposed dominance of the lowest (Gaussian) FC gives a clue to the simple stochastic picture of confinement in agreement with lattice data.
- The $q\bar{q}$ and gluon – gluon interaction is approximately proportional to the charge squared (quadratic Casimir operator $C_D^{(2)}$) which is supported by lattice data.
- Heavy quark dynamics (masses, decay rates etc) calculated via FC are in good agreement with experiment.
- The string profile (field distribution inside the string) is well described by the lowest Gaussian correlator, for both types of probes (see chapter 3).
- The consistency of the model with low-energy theorems is established.
- Lattice calculations yield FC with magnitude and correlation length T_g which quantitatively correspond to the string tension and gluonic condensate known from experiment. The use of lattice measurements allows to put all the method of FC on the firm foundation, since using FC from lattice data enables one to predict hundreds of data: masses, widths, cross sections etc.

- It was predicted that at the deconfinement phase transition the correlator of CE fields D_E vanishes, while that of CM fields D_B stays intact. Lattice measurements have confirmed this and have shown the dramatic decrease of D_E at $T \geq T_c$, while D_B is almost equal to its value at $T = 0$. From this fact it follows that D_E is a good order parameter for the deconfinement phase transition, and also that above T_c one has the phase of "magnetic confinement".

It is worth noting, that the fundamental input of the formalism – the Gaussian correlator – is not a freely adjustable function. Instead, one gets it from the lattice measurements and analytically calculates all the rest without any additional "fitting". It is one of the reasons why so much attention has been paid throughout this review to the question about Gaussian dominance, since in this case one basically does not need any other field-theoretical inputs (like higher correlators).

One should also mention four topics which might have important influence on the development of the method. An important recent development which has a direct bearing on FC is the calculation of the so called gluelump states on the lattice [119] and in the present method [120]. Actually gluelump Green's functions are generalizations of FC and the lowest gluelump states, 1^{--} and 1^{+-} refer directly to the FC of electric and magnetic fields, respectively. Therefore the masses of these gluelump states are simply T_g^{-1} for the corresponding correlators.

The second line of development is the analytic approach with the aim of construction of the equations, defining the FC. In this way one should analytically calculate FC and establish the hierachy of FC corresponding to the Gaussian vacuum. Recently a set of integral equations has been derived in the large N_c limit [70], and the first calculation of T_g in terms of σ was made possible, yielding results in good agreement with lattice data. More extended analysis is necessary in this direction including phase transition and dynamical quarks.

As mentioned in the introduction, the model has also been successfully applied to high energy reactions with small momentum transfer. Here it was possible with the correlator discussed here to calculate the matrix elements of many high energy reactions involving hadrons and (virtual) photons. A discussion of the methods and the results can be found for instance in the reviews [55], [57] and in and the literature quoted there. Also this is a wide field and will be treated separately.

Finally, there is another area of research important for applications and understanding – the interconnection of FC on one hand and OPE and QCD sum rules on another hand, especially from the point of view of OPE violation. As it was argued by one of the authors [133] the problem of OPE violation is tightly connected to a vast and almost unexplored field of interference between purely perturbative and purely nonperturbative contributions. This direction of studies looks promising however difficult. The work on this set of questions [32, 33] in the example of e^-e^+ annihilation raises many interesting problems, especially about ultraviolet renormalons and $1/Q^2$ term, which together with recent lattice measurements of the condensate $\langle FF \rangle$ (see discussion and refs. in [67]) makes this field highly attractive and dynamic – hence still inappropriate for the review.

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Figures

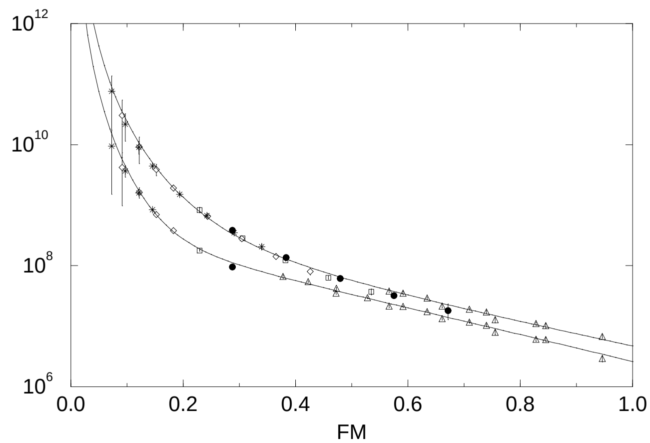


Fig.1. $D_{||}^{lat}/a^4 = D + D_1 + z^2 \partial D_1/\partial z^2$ and $D_\perp/a^4 = D + D_1$ versus z. The lines correspond to the best fit to (2.14) (from [69]).

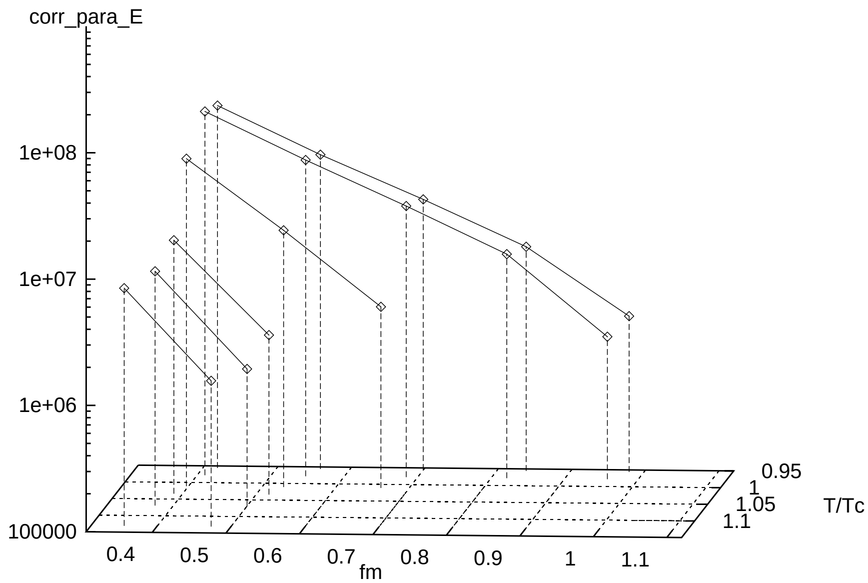


Fig.2. The electric longitudinal correlator versus distance, for different values of T/T_c (from [69]).

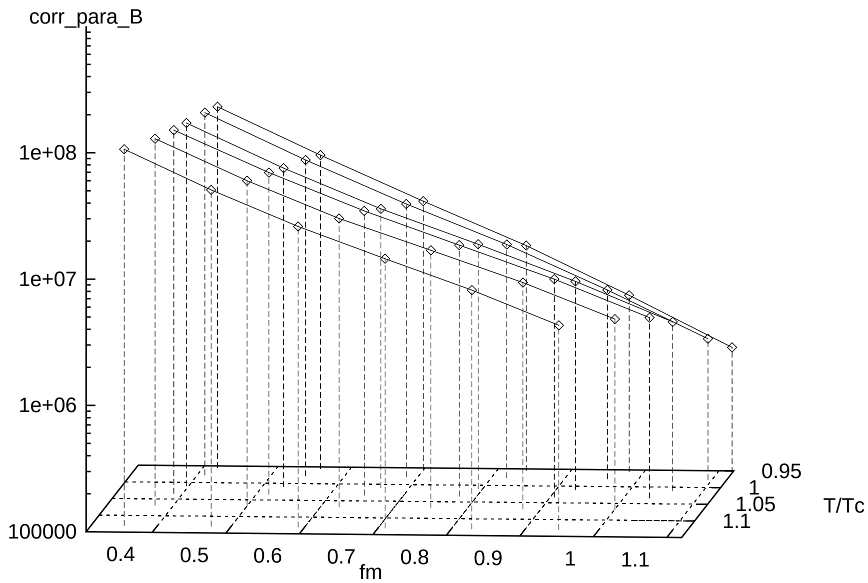


Fig.3. The magnetic longitudinal correlator versus distance, for different values of T/T_c (from [69]).

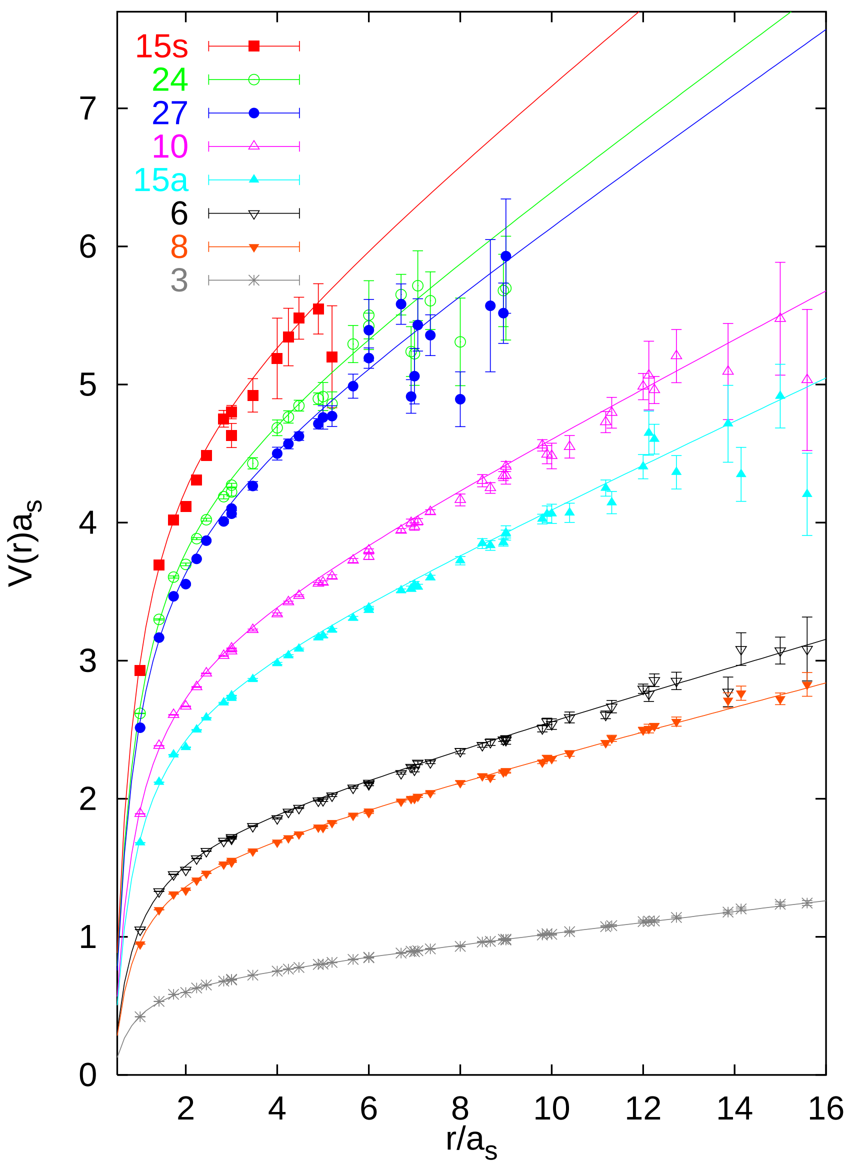


Fig.4. Static potentials for sources in different $SU(3)$ representations (from [102]).

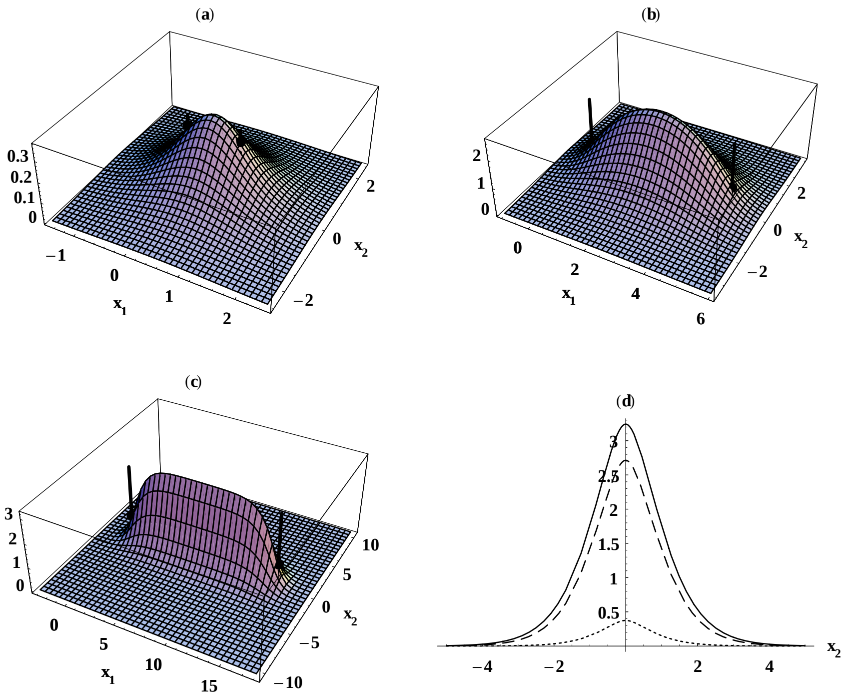
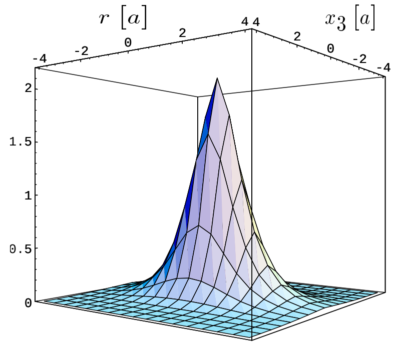
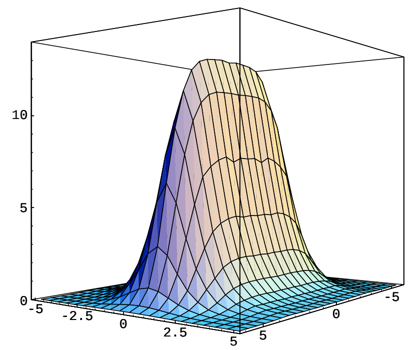


Fig.5. String profiles measured with connected correlators for different distances (from [116]).





Figs.6,7 Difference of the (non-perturbative) squared electric field parallel to the quark (x_3) axis ($-\langle E_z^2(x_3, r)\rangle_{\mathrm{q\bar{q}-vac}}$) in GeV/Fm^3 for quark separation R_W= 0.7 Fm (**Fig.6**, left) and R_W= 3 Fm (**Fig 7**, right); $T_g \equiv a = 0.35$ Fm (from [45]).